Day 60 **IGCSE 10-a-day: Model Solutions**

1 Bounds in geometry

A rectangle has length 7.9 cm and width 4.2 cm, both correct to 1 decimal place.
Work out the lower bound of the area of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side
Length is between 7.85 and 7.95, width is between 4.15 and 4.25
Lower bound of area: use the lower values: $7.85 \times 4.15 = 32.5775$ cm^2
Answer: 32.5775 cm^2

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = 2t^2 + 4t + 4 + \frac{8}{t}$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = 4t + 4 - 8t^{-2}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 4 + 16t^{-3}$

When t = 2: v = 10 and a = 6
Answer: (a) 10 m/s (b) 6 m/s^2

3 Functions that commute

f(x) = 4x + 3 and g(x) = kx − 4, where k is a constant.
fg(x) = gf(x) for all values of x.
Find the value of k.

fg(x) = 4(kx − 4) + 3 = 4kx − 13
gf(x) = k(4x + 3) − 4 = 4kx + 3k − 4
Equate the constant terms: −13 = 3k − 4
3k = −9, so k = −3
Answer: k = −3

4 Rate of change of area or volume

The area, A cm^2, of a circle of radius r cm is $A = \pi r^2$.

(a) Find $\frac{dA}{dr}$ when r = 3.

(b) Find the value of r when $\frac{dA}{dr} = 4\pi$.

$\frac{dA}{dr} = 2\pi r$

When r = 3: $\frac{dA}{dr} = 6\pi$

$2\pi r = 4\pi$, so 2r = 4 and r = 2 (lengths are positive)
Answer: (a) 6π (b) r = 2

5 Iteration

Use the iteration formula $x_{n+1} = \sqrt{5x_n + 11}$ with $x_0 = 1$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{5 \times 1 + 11} = 4.00000...$
$x_2 = \sqrt{5 \times x_1 + 11} = 5.56776...$
$x_3 = \sqrt{5 \times x_2 + 11} = 6.23208...$
Rounded: $x_1 = 4.000$, $x_2 = 5.568$, $x_3 = 6.232$
Answer: $x_1 = 4.000$, $x_2 = 5.568$, $x_3 = 6.232$

6 Maximum revenue

A company sells x thousand items each month. The price of each item is £4 − x.
The monthly revenue is £R thousand.
(a) Show that $R = 4x - x^2$.
(b) Find the maximum monthly revenue, showing that it is a maximum.

$R = x \times (4 - x) = 4x - x^2$

$\frac{dR}{dx} = -2x + 4 = 0$ gives $x = 2$

$\frac{d^2R}{dx^2} = -2 < 0$, so a maximum

$R = 4 \times 2 - 1 \times 2^2 = 4$

Answer: £4 thousand

7 Estimating the median from grouped data

The table gives information about 116 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 5, 34, 64, 88, 116
The median is the value at position 116 ÷ 2 = 58
It is in the class 50 to 70, with 34 values before it, so we need 24 more of the 30 values in this class

Median $\approx 50 + \frac{24}{30} \times 20 = 66$

Answer: 66

8 Range and inverse of a quadratic

$g(x) = x^2 + 4x + 13$ for $x \geq -2$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x + 2)^2 + 9$
$x \geq -2$ includes the minimum point, so $g(x) \geq 9$
(b) $y = (x + 2)^2 + 9$, so $(x + 2)^2 = y - 9$
$x \geq -2$ means $x + 2$ is positive or zero, so $x = -2 + \sqrt{y - 9}$
$g^{-1}(x) = -2 + \sqrt{x - 9}$

Answer: (a) $g(x) \geq 9$ (b) $g^{-1}(x) = -2 + \sqrt{x - 9}$

9 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 4^2 + 4^2 = 32$, so $AM^2 = (AC \div 2)^2 = 8$
$VM^2 = VA^2 - AM^2 = 3^2 - 8 = 1$
VM = 1 cm
(b) Volume $= \frac{1}{3} \times 4^2 \times 1 = \frac{16}{3}$ cm^3

Answer: (a) 1 cm (b) $\frac{16}{3}$ cm^3

10 Counting factors

$N = 2 \times 5^5$

How many factors does N have?

Include 1 and N.

Add 1 to each power and multiply

$(1 + 1) \times (5 + 1) = 2 \times 6 = 12$

Answer: 12