Day 6 **IGCSE 10-a-day: Model Solutions**

1 Solving trigonometric equations

Solve $2\tan x + 5 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = -\frac{5}{2}$

Related acute angle: $\tan^{-1}(\frac{5}{2}) = 68.2°$

tangent is negative in the 2nd and 4th quadrants, so $x = 180° - 68.2°$ and $x = 360° - 68.2°$
$x = 111.8°$ and $x = 291.8°$
Answer: $x = 111.8°$ or $x = 291.8°$

2 Describe a graph transformation

Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of
(a) $y = f(x) + 1$
(b) $y = -f(x)$

(a) Adding 1 to f(x) moves the graph up 1 units
(b) Changing the sign of every y value flips the graph over the x-axis
Answer: (a) A translation by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (b) A reflection in the x-axis

3 Using a line of best fit

The scatter graph shows age of car (x) and value (y).
The equation of the line of best fit is $y = -3x + 37$.
(a) Estimate y when x = 3.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = -3 \times 3 + 37 = 28$
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 28 (b) x = 25 is outside the range of the data

4 Completing the square

(a) Write $x^2 + 6x + 8$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 6x + 8$

(a) Half of 6 is 3, so $x^2 + 6x = (x + 3)^2 - 9$
$x^2 + 6x + 8 = (x + 3)^2 - 9 + 8$
$= (x + 3)^2 - 1$
(b) The minimum is when $x + 3 = 0$, so $x = -3$ and $y = -1$
Answer: (a) $(x + 3)^2 - 1$ (b) $(-3, -1)$

5 Local maximum and minimum values

$f(x) = x^3 - 12x + 2$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 12 = 3(x - 2)(x + 2)$
$f'(x) = 0$ gives $x = -2$ or $x = 2$
$f''(x) = 6x$: $f''(-2) = -12 < 0$ (maximum) and $f''(2) = 12 > 0$ (minimum)
$f(-2) = 18$ and $f(2) = -14$
Answer: Local maximum y = 18, local minimum y = −14

6 Angles of elevation from two points

Points A and B are on level ground, 77 m apart, with B between A and the foot of a tower.
The angle of elevation of the top of the tower is 43° from A and 61° from B.
Work out the height of the tower. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 61° − 43° = 18° (exterior angle of the triangle)

Sine rule in triangle ABT: $\frac{AT}{\sin 119°} = \frac{77}{\sin 18°}$

$AT = \frac{77 \times \sin 61°}{\sin 18°} = 217.94$

Height = AT × sin 43° = 149 m

Answer: 149 m

7 Tangent perpendicular to a line

The tangent at the point P on the curve $y = -x^2 - 3x + 5$ is perpendicular to the line $x - 7y = 6$.
Find the coordinates of P.

The line is $y = \frac{1}{7}x - \frac{6}{7}$, so its gradient is $\frac{1}{7}$

The tangent has gradient −7

$\frac{dy}{dx} = -2x - 3 = -7$ so $x = 2$

$y = -5$

Answer: (2, −5)

8 HCF and LCM

The HCF of two numbers is 9 and their LCM is 270.
One of the numbers is 27.
Work out the other number.

HCF × LCM = product of the two numbers

9 × 270 = 2430

Other number = 2430 ÷ 27 = 90

Answer: 90

9 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 + 2x^2 - 2$ at the point P where $x = -2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 + 4x$, so at P the gradient is 4 and $y = -2$

Tangent: $y = 4x + 6$

Intersection: $x^3 + 2x^2 - 4x - 8 = 0$

$x = -2$ is a repeated root, so the roots are -2, -2 and x_Q

Sum of roots = −2, so $-4 + x_Q = -2$ and $x_Q = 2$

$y = 14$

Answer: (2, 14)

10 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 36}{x^2 - 3x} \div \frac{x^2 - 8x + 12}{x}$

Factorise: $\frac{(x - 6)(x + 6)}{x(x - 3)} \div \frac{(x - 2)(x - 6)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x - 6)(x + 6)}{x(x - 3)} \times \frac{x}{(x - 2)(x - 6)}$

Cancel $(x - 6)$ and x

$= \frac{(x + 6)}{(x - 3)(x - 2)}$

Answer: $\mathbf{\frac{(x + 6)}{(x - 3)(x - 2)}}$