Day 58 **IGCSE 10-a-day: Model Solutions**

1 Probabilities with an unknown

A biased four-sided dice lands on one of four outcomes with these probabilities.
1: 2x; 2: 3x; 3: 0.05; 4: 0.2
(a) Work out the value of x.
(b) The biased four-sided dice is used 80 times. Work out an estimate for the number of times it lands on 1.

(a) Probabilities add up to 1: $2x + 3x + 0.05 + 0.2 = 1$
$5x + 0.25 = 1$, so $5x = 0.75$
$x = 0.15$
(b) $P(1) = 2 \times 0.15 = 0.3$, so $80 \times 0.3 = 24$
Answer: (a) x = 0.15 (b) 24

2 Arithmetic sequences

The 4th term of an arithmetic sequence is 36.
The 8th term of the sequence is 52.
(a) Find an expression for the nth term of the sequence.
(b) Is 104 a term of the sequence? You must show your working.

(a) Common difference = $(52 - 36) \div (8 - 4) = 4$
nth term = $4n + c$. For n = 4: $4 \times 4 + c = 36$, so $c = 20$
nth term = $4n + 20$
(b) Solve $4n + 20 = 104$: $4n = 84$, so $n = 21$
n is a positive whole number, so 104 is the 21st term
Answer: (a) 4n + 20 (b) Yes, it is the 21st term

3 Stationary point of a quartic

$y = -x^4 + 32x$
Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -4x^3 + 32$
$\frac{dy}{dx} = 0$: $-4x^3 + 32 = 0$, so $x^3 = 8$
$x = 2$ and $y = 48$
$\frac{d^2y}{dx^2} = -12x^2$
At $x = 2$: $\frac{d^2y}{dx^2} = -48$, so a maximum
Answer: (2, 48), maximum

4 Standard form in context

The distance from a planet to its star is 1.8×10^{10} km. Light travels at 3×10^5 km/s.
How many seconds does light take to travel this distance? Give your answer in standard form to 3 significant figures. (Calculator)

Time = distance ÷ speed = $(1.8 \times 10^{10}) \div (3 \times 10^5)$
Divide the numbers: 6.0000, and subtract the powers of 10
$= 6.00 \times 10^4$
Answer: 6.00×10^4 seconds

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (4x − 15)°, angle ACB = (x + 30)° and angle CAQ = 49°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
x + 30 = 4x − 15
3x = 45, so x = 15
(b) Angle PAB = 45°
Angles on the straight line PAQ: angle BAC = 180 − 45 − 49 = 86°
Answer: (a) x = 15 (b) 86°

6 Depreciation

A van is bought for £15,000. Each year its value falls by 8%.
After how many whole years is it first worth less than three quarters of its original value? (Calculator)

Target value = £11,250
After 3 years: £11,680.32 (not less)
After 4 years: £10,745.89 (less)
Answer: 4 years

7 Stationary points of y = ax + b/x

$y = -3x - \frac{3}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = -3 + \frac{3}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{3}{x^2} = 3$, so $x^2 = 1$ and $x = 1$ or $x = -1$

$y = -6$ when $x = 1$ and $y = 6$ when $x = -1$

$\frac{d^2y}{dx^2} = -\frac{6}{x^3}$

$x = 1$: $\frac{d^2y}{dx^2} = -6$ so maximum; $x = -1$: $\frac{d^2y}{dx^2} = 6$ so minimum

Answer: (1, −6) maximum; (−1, 6) minimum

8 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-4 - 0}{0 - (-3)} = -\frac{4}{3}$

Perpendicular gradient = $-1 \div (-\frac{4}{3}) = \frac{3}{4}$

M passes through B(−3, 0): $0 = \frac{3}{4} \times (-3) + c$, so $c = \frac{9}{4}$

$y = \frac{3}{4}x + \frac{9}{4}$

Answer: $y = \frac{3}{4}x + \frac{9}{4}$

9 HCF and LCM

The HCF of two numbers is 4 and their LCM is 224.
One of the numbers is 32.
Work out the other number.

HCF × LCM = product of the two numbers
4 × 224 = 896
Other number = 896 ÷ 32 = 28
Answer: 28

10 Stationary point with a 1/x term

A curve has equation $y = 2x^2 - \frac{108}{x}$.

(a) Show that the curve has a stationary point at $x = -3$.
(b) Find the y coordinate of this point and decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 4x + \frac{108}{x^2}$

When $x = -3$: $\frac{dy}{dx} = -12 - (-12) = 0$

$y = 54$

$\frac{d^2y}{dx^2} = 4 - \frac{216}{x^3}$

At $x = -3$: $\frac{d^2y}{dx^2} = 12$, which is positive, so a minimum

Answer: y = 54, minimum