Day 57 **IGCSE 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 5 numbers is 89.
One number is removed and the mean of the remaining 4 numbers is 108.5.
Work out the number that was removed.

Total of the 5 numbers = $5 \times 89 = 445$
Total of the 4 remaining = $4 \times 108.5 = 434$
Number removed = $445 - 434 = 11$
Answer: 11

2 Tangent at a turning point

The graph of $y = -2x^2 + 4x - 7$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -4x + 4$

At P, $\frac{dy}{dx} = 0$, so $-4x + 4 = 0$ and $x = 1$

$y = -5$ at P
The gradient is 0, so the tangent is horizontal: $y = -5$
Answer: $y = -5$

3 Finding a percentage rate

A car was bought for £4,700.
Each year its value decreases by the same percentage.
After 2 years it is worth £3,807.
Work out the percentage decrease each year.

Multiplier for 2 years = £3,807 ÷ £4,700 = 0.81
Yearly multiplier = the square root of 0.81 = 0.9
10% decrease each year
Answer: 10%

4 Increasing or decreasing at a point

$y = 2x^3 - 15x^2 + 36x - 1$
At the point where $x = -3$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 - 30x + 36$

When $x = -3$: $\frac{dy}{dx} = 180$

180 is positive, so the curve is increasing
Answer: increasing (gradient 180)

5 Overall percentage change from repeated changes

A population grows by 6% each year for 3 years.
Work out the overall percentage increase. Give your answer correct to 3 significant figures.

Multiplier = 1 + 6 ÷ 100 = 1.06
Overall multiplier = $1.06^3 = 1.1910$
Percentage increase = $(1.1910 - 1) \times 100 = 19.1\%$
Answer: 19.1%

6 Geometric sequences

The 2nd term of a geometric sequence is 96.
The 4th term of the sequence is 384.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 384 \div 96 = 4$
$r = \sqrt{4} = 2$ (positive, since all terms are positive)
(b) The 2nd term is $a \times r^1$: $a \times 2 = 96$
$a = 96 \div 2 = 48$
Answer: (a) r = 2 (b) 48

7 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and CB = 2 × OA.
$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.
(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.
(b) AM : MB = 1 : 3. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = 2\mathbf{a}$
$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = 2\mathbf{a} + \mathbf{c}$
$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = \mathbf{a} + \mathbf{c}$
(b) $\mathbf{OM} = \mathbf{OA} + \frac{1}{4}\mathbf{AB} = \mathbf{a} + \frac{1}{4}(\mathbf{a} + \mathbf{c})$
$= \frac{5}{4}\mathbf{a} + \frac{1}{4}\mathbf{c}$
Answer: (a) $\mathbf{a} + \mathbf{c}$ (b) $\frac{5}{4}\mathbf{a} + \frac{1}{4}\mathbf{c}$

8 Equations with algebraic fractions

Solve $\frac{2}{x+3} - \frac{4}{x-5} = \frac{3}{2}$

Multiply every term by $(x + 3)(x - 5)$: $2(x - 5) - 4(x + 3) = \frac{3}{2} \times (x + 3)(x - 5)$
Multiply by 2 to clear the fraction: $4(x - 5) - 8(x + 3) = 3(x + 3)(x - 5)$
Expand and rearrange so that one side is 0: $3x^2 - 2x - 1 = 0$
Factorise: $(x - 1)(3x + 1) = 0$
$x = -\frac{1}{3}$ or $x = 1$
Answer: $x = -\frac{1}{3}$ or $x = 1$

9 Minimum surface area of a tank

A skip is a cuboid with a square base of side x metres and height h metres, and has a closed top. Its volume is 125 m^3.
(a) Show that the total outside surface area, A m^2, is $A = 2x^2 + \frac{500}{x}$.
(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 125$, so $h = \frac{125}{x^2}$
$A = 2x^2 + 4xh = 2x^2 + 4x \times \frac{125}{x^2} = 2x^2 + \frac{500}{x}$
$\frac{dA}{dx} = 4x - 500x^{-2} = 0$ gives $x^3 = 125$, so $x = 5$
$\frac{d^2A}{dx^2} = 4 + 1000x^{-3} = 12 > 0$, so a minimum
$A = 150$
Answer: 150 m^2

10 Cone made from a sector

A sector of a circle has radius 18 cm and angle 240°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{240}{360} \times 2\pi \times 18 = 24\pi$ cm

This is the circumference of the base: $2\pi r = 24\pi$, so r = 12 cm

(b) Slant height = 18 cm, so height$^2 = 18^2 - 12^2 = 180$, height = $6\sqrt{5}$ cm

Volume = $\frac{1}{3} \times \pi \times 12^2 \times 6\sqrt{5} = 288\sqrt{5}\pi$ cm^3

Answer: (a) 12 cm (b) $288\sqrt{5}\pi$ cm^3