Day 56 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $2x^4 + 7x^3 + x$
(a) Find f′(x).
(b) Work out the value of f′(−1).

(a) $2x^4 \to 8x^3$, $7x^3 \to 21x^2$, $x \to 1$
f′(x) = $8x^3 + 21x^2 + 1$
(b) f′(−1) = 14
Answer: (a) f′(x) = $8x^3 + 21x^2 + 1$ (b) 14

2 Ordering numbers

Write these numbers in order of size, starting with the largest.
$\frac{1}{5}$, $\frac{7}{11}$, 0.69, 0.66

Convert each to a decimal: $\frac{1}{5} = 0.2$, $\frac{7}{11} = 0.636$, 0.69 = 0.69, 0.66 = 0.66

In order: 0.69, 0.66, $\frac{7}{11}$, $\frac{1}{5}$

Answer: 0.69, 0.66, $\frac{7}{11}$, $\frac{1}{5}$

3 Quadratic inequalities

Solve $x^2 + 2x - 3 \geq 0$

Solve $x^2 + 2x - 3 = 0$: $(x + 3)(x - 1) = 0$, so $x = -3$ or $x = 1$
The graph is a U shape crossing the x-axis at −3 and 1; it is above the axis outside the roots
$x \leq -3$ or $x \geq 1$
Answer: $x \leq -3$ or $x \geq 1$

4 Gradient from a tangent

The diagram shows part of the curve $y = -x^2 + x + 5$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{6 - 4}{1 - (-1)} = 1$

$\frac{dy}{dx} = -2x + 1$

$-2x + 1 = 1$, so $x = 0$
Answer: (a) 1 (b) x = 0

5 Speed-time graphs

The graph shows the speed of a car during 10 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 10 seconds.

(a) Acceleration = change in speed ÷ time = (20 − 0) ÷ 4 = 5 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 20}{2} \times 4 = 40$; rectangle: $20 \times 4 = 80$; triangle: $\frac{1}{2} \times 2 \times 20 = 20$
Total = 140 m, so average speed = 140 ÷ 10 = 14 m/s
Answer: (a) 5 m/s^2 (b) 14 m/s

6 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 47°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 47°
Angle COB is an exterior angle of triangle OAC: 47 + 47 = 94°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 94°

7 Extreme value of an expression in a region

x and y satisfy these inequalities:
$x \geq 0$, $y \geq 1$, $3x + 4y \leq 16$
Find the least possible value of $3x - 3y$.

The region is a triangle. The extreme value occurs at a vertex.
Vertices: (0, 1), (4, 1), (0, 4)
Values of $3x - 3y$: -3, 9, -12
Least value = -12
Answer: -12

8 Two numbers: maximise or minimise

The positive numbers x and y satisfy $3x + y = 18$.
Find the greatest possible value of xy, showing that it is the greatest.

$y = 18 - 3x$, so $xy = x(18 - 3x) = -3x^2 + 18x$
$\frac{d}{dx}(xy) = -6x + 18 = 0$ gives $x = 3$
Second derivative = $-6 < 0$, so a maximum
$y = 9$, so the greatest value of xy is $3 \times 9 = 27$
Answer: 27

9 Combined and changing means

The mean mark of 9 students in the boys is 61.
The mean mark of 6 students in the girls is 69.
Work out the mean mark of all 15 students.

Total marks for the boys = $9 \times 61 = 549$
Total marks for the girls = $6 \times 69 = 414$
Mean = $(549 + 414) \div 15 = 64.2$
Answer: 64.2

10 Standard form

Work out $(3.6 \times 10^8) - (5.4 \times 10^7)$
Give your answer in standard form.

Write the first number as a multiple of 10^7: $3.6 \times 10^8 = 36 \times 10^7$
$(36 - 5.4) \times 10^7 = 30.6 \times 10^7$
$= 3.06 \times 10^8$
Answer: 3.06×10^8