Day 55 **IGCSE 10-a-day: Model Solutions**

1 Two picks without replacement

A bag contains 3 green and 6 black cubes.
Two cubes are taken at random without replacement.
Work out the probability that the two cubes are the same colour.

P(same colour) = sum over each colour of P(both that colour)

$\frac{3}{9} \times \frac{2}{8} + \frac{6}{9} \times \frac{5}{8}$

$= \frac{1}{2}$

Answer: $\mathbf{\frac{1}{2}}$

2 Negative powers

Differentiate $-5x^2 + 2x - \frac{7}{x^2}$ with respect to x.

Write negative powers as indices: $y = -5x^2 + 2x - 7x^{-2}$

$-5x^2 \rightarrow -10x$, $2x \rightarrow 2$, $-7x^{-2} \rightarrow 14x^{-3}$

$\frac{dy}{dx} = -10x + 2 + 14x^{-3}$

Answer: $\mathbf{\frac{dy}{dx} = -10x + 2 + \frac{14}{x^3}}$

3 Quadratic formula and surds

Solve $3x^2 - 8x + 3 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 3, b = −8, c = 3

$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 3 \times 3}}{2 \times 3}$

$x = \frac{8 \pm \sqrt{28}}{6}$

$\sqrt{28} = 2\sqrt{7}$, so $x = \frac{8 \pm 2\sqrt{7}}{6}$

Divide the top and bottom by 2: $x = \frac{4 \pm \sqrt{7}}{3}$

Answer: $\mathbf{x = \frac{4 \pm \sqrt{7}}{3}}$

4 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 13^2 - 5^2 = 144$
AM = 12 cm
AB = 2 × 12 = 24 cm

(b) sin(angle AOM) = $\frac{12}{13}$

Angle AOM = 67.38°, so angle AOB = 2 × 67.38 = 134.8°

Answer: (a) 24 cm (b) 134.8°

5 Nested surds

Show that $\sqrt{37 + 20\sqrt{3}} = 5 + 2\sqrt{3}$

Square the right-hand side: $(5 + 2\sqrt{3})^2 = 25 + 20\sqrt{3} + 12$

$= 37 + 20\sqrt{3}$

So $\sqrt{37 + 20\sqrt{3}} = 5 + 2\sqrt{3}$

Answer: $\sqrt{37 + 20\sqrt{3}} = 5 + 2\sqrt{3}$

6 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = -2t^3 + 24t^2 + 4t$ for $t \geq 0$

Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -6t^2 + 48t + 4$

$a = \frac{dv}{dt} = -12t + 48$

Maximum v when $a = 0$: $-12t + 48 = 0$, so $t = 4$

$\frac{d^2v}{dt^2} = -12 < 0$, so v is a maximum

$v = 100$

Answer: 100 m/s when t = 4 s

7 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the times, in minutes, taken by 40 runners to finish a race.

(a) Find the median. (b) Find the interquartile range.

(c) How many runners took more than 20?

(a) Median at cumulative frequency 20: reading across gives 30

(b) Lower quartile at 10 gives 20, upper quartile at 30 gives 40

IQR = 40 − 20 = 20

(c) At 20 the cumulative frequency is 10, so 40 − 10 = 30

Answer: (a) 30 (b) 20 (c) 30

8 Area of a segment

The circle has centre O and radius 10 cm. A and B are points on the circle and angle AOB = 100°. (Calculator)

Work out the area of the shaded segment. Give your answer correct to 3 significant figures.

The diagram is not drawn accurately.

Area of sector OAB = $\frac{100}{360} \times \pi \times 10^2 = 87.266$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 10 \times 10 \times \sin 100° = 49.240$ cm^2

Segment = sector − triangle = 87.266 − 49.240 = 38.026

= 38.0 cm^2 (3 s.f.)

Answer: 38.0 cm^2

9 Tangent meets the curve again

A curve has equation $y = x^3 - 2x^2 - 9x + 5$.

The tangent to the curve at the point A, where x = 3, meets the curve again at the point B.

Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 4x - 9$, so the gradient at A is 6, and A = (3, −13)

Tangent: $y = 6x - 31$

Equate: $x^3 - 2x^2 - 15x + 36 = 0$, which has a repeated root x = 3

Sum of the roots = 2, so 3 + 3 + x = 2 and x = −4

y = −55

Answer: B (−4, −55)

10 Probability and quadratic equations

A bag contains some beads. 6 of them are blue and the rest are black.
Two beads are taken at random without replacement.

The probability that both are blue is $\frac{3}{11}$.

Work out how many beads are black.

Let there be n beads altogether

P(both blue) = $\frac{6}{n} \times \frac{5}{n-1} = \frac{30}{n(n-1)}$

$\frac{30}{n(n-1)} = \frac{3}{11}$, so $n(n-1) = 110$, giving $n^2 - n - 110 = 0$

$(n-11)(n+10) = 0$, so $n = 11$ (n cannot be negative)

Number of beads that are black = $11 - 6 = 5$

Answer: 5