Day 54 **IGCSE 10-a-day: Model Solutions**

1 Direct variation

P is directly proportional to the square of t.
When t = 10, P = 600.
(a) Find a formula for P in terms of t.
(b) Find P when t = 6.

$P = kt^2$

$600 = k \times 10^2$ so $k = 6$

$P = 6t^2$

$P = 6 \times 36 = 216$

Answer: (a) $P = 6t^2$ (b) $P = 216$

2 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 53°.
B is a point on the same line, 25 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 41°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 53° = \frac{h}{x}$, so $x = h \div \tan 53°$

From B: $\tan 41° = \frac{h}{x + 25}$, so $x + 25 = h \div \tan 41°$

Subtract: $25 = h \div \tan 41° - h \div \tan 53° = h(1.1504 - 0.75355)$
$h = 25 \div 0.39681 = 63.0$ m (1 d.p.)
Answer: 63.0 m

3 Tests of divisibility

The 5-digit number 66k46 is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 6 + 6 + 4 + 6 + k = 22 + k
This must be a multiple of 9, so k = 5
Answer: k = 5

4 Gradient with fractional or negative powers

The equation of a curve is $y = 4x^2 - 2x - \frac{10}{x}$.

Work out the gradient of the curve at the point where x = 2.

$\frac{dy}{dx} = 8x - 2 + 10x^{-2}$

When x = 2: gradient = $\frac{33}{2}$

Answer: $\frac{33}{2}$

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{16^2 + 11^2 - 16^2}{2 \times 16 \times 11} = \frac{121}{352}$

$\theta = 69.9°$ (1 d.p.)

(b) Area $= \frac{1}{2} \times 16 \times 11 \times \sin 69.9°$

$= 82.6\ \text{cm}^2$ (3 s.f.)

Answer: (a) 69.9° (b) 82.6 cm^2

6 Equation of a circle

A circle has centre C(−4, 3) and passes through the point P(−2, −6).
Find an equation of the circle.

$r^2 = CP^2 = (-2 - (-4))^2 + (-6 - 3)^2$
$= 4 + 81 = 85$
$(x + 4)^2 + (y - 3)^2 = 85$

Answer: $(x + 4)^2 + (y - 3)^2 = 85$

7 Reading a trigonometric graph

The diagram shows the graph of $y = a \cos bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 2, least value = 0
c = ½(2 + 0) = 1 and a = ½(2 − 0) = 1
One full cycle takes 180° (2 cycles between 0 and 360), so b = 360 ÷ 180 = 2

Answer: a = 1, b = 2, c = 1

8 Minimum surface area of a tank

A tank is a cuboid with a square base of side x metres and height h metres, and has a closed top. Its volume is 343 m^3.

(a) Show that the total outside surface area, A m^2, is $A = 2x^2 + \frac{1372}{x}$.

(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 343$, so $h = \frac{343}{x^2}$

$A = 2x^2 + 4xh = 2x^2 + 4x \times \frac{343}{x^2} = 2x^2 + \frac{1372}{x}$

$\frac{dA}{dx} = 4x - 1372x^{-2} = 0$ gives $x^3 = 343$, so $x = 7$

$\frac{d^2A}{dx^2} = 4 + 2744x^{-3} = 12 > 0$, so a minimum

$A = 294$

Answer: 294 m^2

9 Mixtures and percentage profit

A shop mixes two types of flour.
Type A costs £11.00 per kg and type B costs £5.00 per kg.
They are mixed in the ratio A : B = 1 : 3.
The mixture is sold for £7.80 per kg.
Work out the percentage profit.

Cost of 4 kg of mixture = 1 × £11.00 + 3 × £5.00 = £26.00
Cost per kg = £26.00 ÷ 4 = £6.50
Profit per kg = £7.80 − £6.50 = £1.30
Percentage profit = 1.30 ÷ 6.50 × 100 = 20%
Answer: 20%

10 Rectangle under a parabola

The diagram shows the curve $y = 6 - 2x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).
(a) Show that the area of ABCD is $-4x^3 + 12x$.
(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $6 - 2x^2$, so area = $2x(6 - 2x^2) = -4x^3 + 12x$
$\frac{dA}{dx} = -12x^2 + 12 = 0$, so $x^2 = 1$ and $x = 1$ $(x > 0)$
$\frac{d^2A}{dx^2} = -24x < 0$ when $x = 1$, so a maximum
Area = $12 \times 1 - 4 \times 1^3 = 8$
Answer: 8