Day 52 **IGCSE 10-a-day: Model Solutions**

1 Quadratic inequalities

Solve $x^2 \geq 121$

$x^2 \geq 121$
Critical values: $x = \pm 11$
Sketch the U-shaped curve: the expression is positive or zero outside the critical values
Answer: $x \leq -11$ or $x \geq 11$

2 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 33 is taken. Work out how many come from Year 8.
(b) A different stratified sample has 21 from Year 7. Work out the size of this sample.

Total = 175 + 175 + 300 + 175 = 825

(a) Year 8: $\frac{175}{825} \times 33 = 7$

(b) Fraction sampled = $21 \div 175 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 825 = 99$

Answer: (a) 7 (b) 99

3 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = t^3 - 12t^2 + 45t + 2$ $(t \geq 0)$
Find the times when P is at rest.

$v = \frac{ds}{dt} = 3t^2 - 24t + 45$

P is at rest when $v = 0$: $3t^2 - 24t + 45 = 0$
$3(t - 3)(t - 5) = 0$, so $t = 3$ or $t = 5$
Answer: $t = 3$ and $t = 5$

4 Simultaneous equations

Solve the simultaneous equations
$-x - y = 9$
$3x - 5y = 13$

To match the y terms: (1) × 5 gives $-5x - 5y = 45$
Subtract the equations to eliminate y: $-8x = 32$
$x = -4$
Substitute $x = -4$ into (1): $-y + 4 = 9$
$-y = 5$, so $y = -5$
Answer: $x = -4$, $y = -5$

5 Product of brackets: tangent gradient

$y = x(x - 1)(3x - 3)$
Work out the gradient of the curve at the point where $x = -2$.

Expand: $y = 3x^3 - 6x^2 + 3x$
$\frac{dy}{dx} = 9x^2 - 12x + 3$

$x = -2$: $\frac{dy}{dx} = 63$

Answer: 63

6 Conditional probability from a table

The table shows information about 113 employees.
(a) An employee who walks to work is chosen at random. Work out the probability that this employee is a man.
(b) Two of the 113 employees are chosen at random. Work out the probability that both are men.

(a) 15 employees are in the Walk column, and 7 of them are men

Probability = $\frac{7}{15}$

(b) There are 64 men, so P(both) = $\frac{64}{113} \times \frac{63}{112} = \frac{36}{113}$

Answer: (a) $\frac{7}{15}$ (b) $\frac{36}{113}$

7 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, 1) lies on the curve and the gradient of the curve at this point is 5.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$

Point on curve: $a + b = 1$. Gradient: $2a - b = 5$
Solving simultaneously gives $a = 2$ and $b = -1$
Answer: $a = 2$, $b = -1$

8 Percentages with an unknown rate

The price of a ticket is increased by x%.
The new price is then decreased by x%.
The final price is 36% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 36% less, so the multiplier is 0.64: $1 - \frac{x^2}{10000} = 0.64$

$\frac{x^2}{10000} = 0.36$, so $x^2 = 3600$ and $x = 60$
Answer: $x = 60$

9 Compound interest: find the rate

£8,000 is invested for 3 years at r% per year compound interest.
At the end of 3 years the investment is worth £10,077.70.
Work out the value of r. (Calculator)

$8000 \times (1 + \frac{r}{100})^3 = £10{,}077.70$

$(1 + \frac{r}{100})^3 = 10077.7 \div 8000 = 1.2597125$

$1 + \frac{r}{100} = 1.08$

$r = 8$
Answer: $r = 8$

10 Pythagoras with algebra

ABC is a right-angled triangle with sides $(x - 1)$ cm, (x) cm and $(x + 1)$ cm. The longest side is $(x + 1)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x - 1)^2 + x^2 = (x + 1)^2$
Expand and collect terms: $x^2 - 4x = 0$
Factorise: $x(x - 4) = 0$
$x = 4$ or $x = 0$, and x must be positive, so $x = 4$
(b) The two shorter sides are 3 cm and 4 cm
Area $= \frac{1}{2} \times 3 \times 4 = 6$ cm^2

Answer: (a) x = 4 (b) 6 cm^2