Day 51 **IGCSE 10-a-day: Model Solutions**

1 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 58° and angle CAQ = 55°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 55° (alternate segment theorem)
(b) Angle ACB = angle PAB = 58° (alternate segment theorem)
Answer: (a) 55° (b) 58°

2 Mean of a changing set

The mean of 9 numbers is 60.
A new number is added and the mean of all 10 numbers is 57.6.
Work out the new number.

Total of the 9 numbers = 9 × 60 = 540
New total = 10 × 57.6 = 576
New number = 576 − 540 = 36
Answer: 36

3 Where is the second derivative zero?

$y = -x^3 - 9x^2 - x - 1$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = -3x^2 - 18x - 1$

$\frac{d^2y}{dx^2} = -6x - 18$

$\frac{d^2y}{dx^2} = 0$ gives $-6x - 18 = 0$, so $x = -3$

$\frac{dy}{dx} = 26$ when $x = -3$

Answer: (a) $-6x - 18$ (b) $x = -3$, gradient 26

4 Error intervals

The capacity of a tank is 1.4 litres, truncated to 1 decimal place.
Write down the error interval for the capacity of a tank, using c.

Truncating cuts off the later digits without rounding.
The smallest possible value is 1.4 and the value must be less than 1.5
$1.4 \leq c < 1.5$
Answer: $1.4 \leq c < 1.5$

5 Increasing and decreasing intervals

$y = -2x^3 - 9x^2 - 12x + 2$
Find the range of values of x for which y is decreasing.

$\frac{dy}{dx} = -6x^2 - 18x - 12 = -6(x + 2)(x + 1)$

$\frac{dy}{dx} = 0$ when $x = -2$ and $x = -1$

The graph of $\frac{dy}{dx}$ against x is a ∩-shaped quadratic

y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$

So y is decreasing when $x < -2$ or $x > -1$
Answer: $x < -2$ or $x > -1$

6 Fractional indices

Given that $x > 0$, solve $x^{\frac{2}{3}} = 16$

Raise both sides to the power $\frac{3}{2}$

$x = (16)^{\frac{3}{2}}$
$x = 64$
Answer: $x = 64$

7 Overall percentage change from repeated changes

The price of a coat is decreased by 20% and then increased by 10%.
Work out the overall percentage change. State whether it is an increase or a decrease.

Multipliers: 0.8 and 1.1
Overall multiplier = $0.8 \times 1.1 = 0.88$
12% decrease
Answer: 12% decrease

8 Curves that touch

The curves $y = x^2 + 5x + 3$ and $y = -x^2 + 17x - 15$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $2x^2 - 12x + 18 = 0$
That is $2(x - 3)^2 = 0$ with a repeated root $x = 3$
$y = 27$, so the point is (3, 27)
$\frac{dy}{dx} = 2x + 5$ on the first curve: gradient 11
Tangent: $y = 11x - 6$
Answer: (3, 27), $y = 11x - 6$

9 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x + 6)$ cm, $(x - 1)$ cm and $(x + 7)$ cm.
The longest side is $(x + 7)$ cm.
(a) Show that $x^2 - 4x - 12 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 6)^2 + (x - 1)^2 = (x + 7)^2$
Expand: $x^2 + 12x + 36 + x^2 - 2x + 1 = x^2 + 14x + 49$
Rearrange: $x^2 - 4x - 12 = 0$
(b) Factorise: $(x - 6)(x + 2) = 0$, so $x = 6$ (the other root, $x = -2$, does not give valid side lengths)
The shorter sides are 12 cm and 5 cm, so area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2

Answer: (b) 30 cm^2

10 Linear programming in context

A workshop makes x tables and y chairs each week.
Each table needs 6 hours of work and 3 metres of wood. Each chair needs 2 hours of work and 6 metres of wood.
There are 48 hours of work and 54 metres of wood available.
The income is £7 for each table and £4 for each chair.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest income.

(a) $6x + 2y \leq 48$ and $3x + 6y \leq 54$
(b) Vertices of the feasible region: (0, 0), (0, 9), (6, 6), (8, 0)
Income = $7x + 4y$: 0, 36, 66, 56
Greatest income = £66 with $x = 6$, $y = 6$
Answer: (a) $6x + 2y \leq 48$, $3x + 6y \leq 54$ (b) £66