Day 50 **IGCSE 10-a-day: Model Solutions**

1 Exact trigonometric values

Work out the exact value of $4 \cos 60° \times \cos 60°$.
Give your answer in its simplest form, with no decimals.

$\cos 60° = \frac{1}{2}$ and $\cos 60° = \frac{1}{2}$

Substitute and simplify (rationalise any denominator)
$= 1$

Answer: 1

2 Angles in polygons

Each interior angle of a regular polygon is 150°.
Work out the number of sides of the polygon.

Exterior angle $= 180° - 150° = 30°$
Number of sides $= 360° \div 30° = 12$

Answer: 12

3 Trigonometric equations

Solve $4 \cos x° - 2 = 2$ for $0 \leq x \leq 360$

Rearrange: $4 \cos x° = 4$, so $\cos x° = 1$
From the graph of $y = \cos x°$, $\cos x° = 1$ when $x = 0$ or 360

Answer: x = 0, 360

4 Second derivative and nature at a given point

$y = -2x^3 - 3x^2 - 6$
Use the second derivative to show that the curve has a maximum at $x = 0$.

$\frac{dy}{dx} = -6x^2 - 6x$

When $x = 0$: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = -12x - 6$

When $x = 0$: $\frac{d^2y}{dx^2} = -6$, which is negative

So there is a maximum at $x = 0$

Answer: maximum at x = 0

5 Surds

Given that $\sqrt{72} - \sqrt{2} = \sqrt{n}$,
find the value of n.

$\sqrt{72} = 6\sqrt{2}$
$6\sqrt{2} - \sqrt{2} = 5\sqrt{2}$
$5\sqrt{2} = \sqrt{50}$, so $n = 50$

Answer: n = 50

6 Inverse proportion

T is inversely proportional to n.
When n = 4, T = 5.
Find the positive value of n when T = 4.

$T = \frac{k}{n}$

$5 = \frac{k}{4}$, so $k = 5 \times 4 = 20$

$4 = \frac{20}{n}$, so $n = \frac{20}{4} = 5$

Answer: n = 5

7 Compound interest: how many years

£10,000 is invested at 4% per year compound interest.
Work out the least whole number of years after which the investment is worth more than £12,500. (Calculator)

Value after n years = 10000×1.04^n
After 5 years: £12,166.53 (less than £12,500)
After 6 years: £12,653.19 (more than £12,500)
Answer: 6 years

8 Turning point of a quadratic by differentiation

$f(x) = 3x^2 + 12x - 2$
Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = 6x + 12$
$f'(x) = 0$: $6x + 12 = 0$, so $x = -2$
$f(-2) = -14$
$f''(x) = 6$, which is positive, so it is a minimum
Answer: $(-2, -14)$, minimum

9 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 16^2 + 16^2 = 512$, so $AM^2 = (AC \div 2)^2 = 128$
$VM^2 = VA^2 - AM^2 = 12^2 - 128 = 16$
VM = 4 cm

(b) Volume $= \frac{1}{3} \times 16^2 \times 4 = \frac{1024}{3}$ cm^3

Answer: (a) 4 cm (b) $\frac{1024}{3}$ cm^3

10 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + 2x + 3$ at the point P where x = 0 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 2$: at P, gradient = 2 and y = 3

Normal: $y = -\frac{1}{2}x + 3$

Intersection: $x^2 + \frac{5}{2}x = 0$

One root is x = 0, and the sum of the roots is $-\frac{5}{2}$

So $x_Q = -\frac{5}{2} - 0 = -\frac{5}{2}$

$y = \frac{17}{4}$

Answer: $(-\frac{5}{2}, \frac{17}{4})$