Day 5 **IGCSE 10-a-day: Model Solutions**

1 Coordinates and gradient at the y-intercept

$y = 3x^4 - x^2 - x + 4$

The curve crosses the y-axis at the point A.

Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 4, so A is (0, 4)

$\frac{dy}{dx} = 12x^3 - 2x - 1$

When x = 0, $\frac{dy}{dx} = -1$

Answer: A = (0, 4), gradient −1

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 70°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angles in quadrilateral OAPB: 360 − 90 − 90 − 70 = 110°

(b) The angle at the circumference is half the angle at the centre: 110 ÷ 2 = 55°

Answer: (a) 110° (b) 55°

3 Stationary points of a quartic

A curve has equation $y = 2x^4 - 4x^2 + 8$.

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 8x = 8x(x - 1)(x + 1)$

$\frac{dy}{dx} = 0$ gives x = 0, x = 1 or x = −1

y = 8 when x = 0, and y = 6 when x = ±1

$\frac{d^2y}{dx^2} = 24x^2 - 8$: it is −8 at x = 0 (maximum) and 16 at x = ±1 (minimum)

Answer: (0, 8) maximum; (1, 6) and (−1, 6) minima

4 Trigonometric equations

Solve $4 \tan x° + 2 = -2$ for $0 \leq x \leq 360$

Rearrange: $4 \tan x° = -4$, so $\tan x° = -1$

The acute angle with tangent 1 is 45°; tangent is negative in the second and fourth quadrants

x = 135 or x = 315 (180 − 45 and 360 − 45)

Answer: x = 135, 315

5 Show that dy/dx = ...

$y = \frac{(3x + 5)(x + 4)}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{3x^2 - 20}{x^2}$.

(b) Work out the gradient of the curve when x = 2.

Expand the brackets: $(3x + 5)(x + 4) = 3x^2 + 17x + 20$, then divide each term by x: $y = 3x + 17 + 20x^{-1}$

$\frac{dy}{dx} = 3 - \frac{20}{x^2}$

$= \frac{3x^2 - 20}{x^2}$

When x = 2: $\frac{dy}{dx} = -2$

Answer: (b) −2

6 Equation of a transformed graph

$f(x) = x^2 - 2x - 3$

The graph of $y = f(x)$ is transformed. Find the equation of the graph after it is

(a) stretched parallel to the y-axis with scale factor 3

(b) reflected in the y-axis

(a) $y = 3f(x)$: multiply every term by 3. So $y = 3x^2 - 6x - 9$

(b) $y = f(-x)$: replace x by $-x$. So $y = x^2 + 2x - 3$

Answer: (a) $y = 3x^2 - 6x - 9$ (b) $y = x^2 + 2x - 3$

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.

(a) Find an estimate for the median and the interquartile range.

(b) Estimate how many of the 80 had a mass greater than 25.

(a) Median at cumulative frequency 40, so median = 50

Lower quartile at 20, so LQ = 40. Upper quartile at 60, so UQ = 60

Interquartile range = 60 − 40 = 20

(b) At 25, cumulative frequency = 5

Number greater = 80 − 5 = 75

Answer: (a) median 50, IQR 20 (b) 75

8 nth term of a cubic sequence

Here are the first five terms of a sequence.

−6, −13, −32, −69, −130

The nth term is of the form $an^3 + bn + c$.

Find the values of a, b and c.

1st differences: −7, −19, −37, −61. 2nd: −12, −18, −24. 3rd: −6, −6

Third differences = 6a = −6 so a = −1

Subtract $-n^3$: −5, −5, −5, −5, −5

This is linear with common difference 0, so b = 0, and c = −5 − 0 = −5

Answer: a = −1, b = 0, c = −5

9 Bounds of roots

The volume of a cube is 745 cm^3, correct to the nearest cm^3.

Work out the lower bound of the edge length of the cube.

Give your answer correct to 3 significant figures. (Calculator)

$744.5 \leq$ volume < 745.5

Lower bound of edge = $\sqrt[3]{744.5}$

= 9.06334... = 9.06 (3 s.f.)

Answer: 9.06 cm

10 Algebraic proof (harder)

Show algebraically that the difference between the cubes of any two consecutive integers is always an odd number.

Let the integers be k and k + 1.

$(k + 1)^3 - k^3 = k^3 + 3k^2 + 3k + 1 - k^3 = 3k^2 + 3k + 1$

$= 3k(k + 1) + 1$

k(k + 1) is even, so 3k(k + 1) is even, and adding 1 gives an odd number

Answer: Proved