Day 49 **IGCSE 10-a-day: Model Solutions**

1 Linear programming in context

A farm grows x hectares of wheat and y hectares of barley.
Each hectare of wheat needs 1 hour of labour and 6 kg of seed. Each hectare of barley needs 3 hours of labour and 3 kg of seed.
There are 36 hours of labour and 66 kg of seed available.
The profit is £5 for each hectare of wheat and £8 for each hectare of barley.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $x + 3y \leq 36$ and $6x + 3y \leq 66$
(b) Vertices of the feasible region: (0, 0), (0, 12), (6, 10), (11, 0)
Profit = 5x + 8y: 0, 96, 110, 55
Greatest profit = £110 with x = 6, y = 10
Answer: (a) $x + 3y \leq 36$, $6x + 3y \leq 66$ (b) £110

2 Expand then differentiate

$y = (x + 6)(x + 3)$
Find $\frac{dy}{dx}$.

Expand: $y = x^2 + 9x + 18$
$x^2 \rightarrow 2x$, $9x \rightarrow 9$
$\frac{dy}{dx} = 2x + 9$
Answer: $\frac{dy}{dx} = 2x + 9$

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 47°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 47 = 94°$
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = $(180 - 94) \div 2 = 43°$
Answer: (a) 94° (b) 43°

4 Parallel vectors

A = (−2, 3), B = (6, −2), C = (−2, 2) and D = (k, −8).
AB is parallel to **CD**.
Find the value of k.

$\mathbf{AB} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$ and $\mathbf{CD} = \begin{pmatrix} k + 2 \\ -10 \end{pmatrix}$
Vertical parts: $-10 = \lambda \times (-5)$, so $\lambda = 2$
Horizontal: $k + 2 = 2 \times 8 = 16$
$k = 14$
Answer: $k = 14$

5 Index equations

Solve $3^{4x - 1} = 27^{2x + 5}$

Write both sides as powers of 3: $27 = 3^3$
Equate the powers: $4x - 1 = 6x + 15$
$-2x = 16$, so $x = -8$
Answer: $x = -8$

6 Ratio and money transferred

Zane and Leila share some money in the ratio 4 : 5.
Zane gives Leila £2.
The ratio of their money is now 1 : 2.
How much money do they have altogether?

Let the amounts be 4k and 5k, so the total is 9k

Zane now has $4k - 2$, which is $\frac{1}{3}$ of the total

$3(4k - 2) = 1 \times 9k$
$12k - 6 = 9k$, so $3k = 6$ and $k = 2$
Total = $9 \times 2 = £18$
Answer: £18

7 Gradient function from a given point

A curve has equation $y = -2x^3 - 15x^2 - 36x - 8$.
The point A (2, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute x = 2 into the equation of the curve
$y = -156$ when $x = 2$, so $k = -156$
$\frac{dy}{dx} = -6x^2 - 30x - 36$
At A: gradient = -120
Answer: k = -156, gradient = -120

8 Reverse percentages

The price of a watch is increased by 15%.
Later the new price is reduced by 20% in a sale.
The sale price is £469.20.
Work out the price before the increase. (Calculator)

Multipliers: 1.15 and 0.8
Overall multiplier = $1.15 \times 0.8 = 0.92$
Original $\times$ 0.92 = £469.20
Original = £469.20 ÷ 0.92 = £510
Answer: £510

9 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = 2x^2 - 5x - 3$.
Find the value of k.

Equate: $2x^2 - 5x - 3 = -2x + k$
Rearrange: $2x^2 - 3x - 3 - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $9 - 4 \times 2 \times (-3 - k) = 0$
$8k + 33 = 0$
$k = -\frac{33}{8}$

Answer: k = $-\frac{33}{8}$

10 Where two tangents meet

The curve C has equation $y = 2x^2 + 3x + 1$.
The points A and B on C have x coordinates -3 and 0.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 4x + 3$. At A: gradient -9, A = $(-3, 10)$

Tangent at A: $y = -9x - 17$. At B: gradient 3, B = (0, 1)

Tangent at B: $y = 3x + 1$

Equate: $-9x - 17 = 3x + 1$, so $x = -\frac{3}{2}$ and $y = -\frac{7}{2}$

Answer: T $\left(-\frac{3}{2}, -\frac{7}{2}\right)$