Day 48 **IGCSE 10-a-day: Model Solutions**

1 Exact trigonometric values

Work out the exact value of $\sin^2 60° + \tan^2 45°$.
Give your answer in its simplest form, with no decimals.

$\sin 60° = \frac{\sqrt{3}}{2}$ and $\tan 45° = 1$

Square them: $\frac{3}{4}$ and 1

Combine: $\frac{7}{4}$

Answer: $\mathbf{\frac{7}{4}}$

2 Gradient with fractional or negative powers

The equation of a curve is $y = -4x^{3/2} + 2\sqrt{x}$.
Work out the gradient of the curve at the point where $x = 1$.

$\frac{dy}{dx} = -6\sqrt{x} + x^{-1/2}$

When $x = 1$: gradient $= -5$

Answer: $\mathbf{-5}$

3 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P(24, −7) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{7}{24}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{7}{24}) = \frac{24}{7}$

$y - (-7) = \frac{24}{7}(x - 24)$

Multiply out and rearrange: $24x - 7y = 625$

(b) On the x-axis $y = 0$, so $24x = 625$

$x = \frac{625}{24}$, so T is $(\frac{625}{24}, 0)$

Answer: (a) $24x - 7y = 625$ (b) T = $(\frac{625}{24}, 0)$

4 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the product is greater than 8.
(b) Given that at least one number is 3, work out the probability that the product is greater than 8.

(a) There are $4 \times 6 = 24$ equally likely outcomes, and 10 of them satisfy: the product is greater than 8

Probability = $\frac{5}{12}$

(b) 9 outcomes satisfy: at least one number is 3. Of these, 5 also satisfy: the product is greater than 8

Probability = $\frac{5}{9}$

Answer: (a) $\frac{5}{12}$ (b) $\frac{5}{9}$

5 Tangent to a curve with negative or fractional powers

A curve has equation $y = 3x^2 - \frac{6}{x}$.

Find the equation of the tangent to the curve at the point where $x = -2$.

$\frac{dy}{dx} = 6x + 6x^{-2}$

At $x = -2$: $y = 15$ and gradient $= -\frac{21}{2}$

$y - 15 = -\frac{21}{2}(x + 2)$

$y = -\frac{21}{2}x - 6$

Answer: $y = -\frac{21}{2}x - 6$

6 Inverse proportion

y is inversely proportional to x.
When x = 5, y = 12.
Find the positive value of x when y = 20.

$y = \frac{k}{x}$

$12 = \frac{k}{5}$, so $k = 12 \times 5 = 60$

$20 = \frac{60}{x}$, so $x = \frac{60}{20} = 3$

Answer: x = 3

7 Linear programming in context

A bakery makes x cakes and y loaves in a morning.
Each cake needs 6 minutes of oven time and 6 kg of flour. Each loaf needs 3 minutes of oven time and 6 kg of flour.
There are 30 minutes of oven time and 48 kg of flour available.
The profit is £4 for each cake and £3 for each loaf.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $6x + 3y \leq 30$ and $6x + 6y \leq 48$
(b) Vertices of the feasible region: (0, 0), (0, 8), (2, 6), (5, 0)
Profit = 4x + 3y: 0, 24, 26, 20
Greatest profit = £26 with x = 2, y = 6
Answer: (a) $6x + 3y \leq 30$, $6x + 6y \leq 48$ (b) £26

8 HCF and LCM

The HCF of two numbers is 6 and their LCM is 330.
The smaller number is 30.
Work out the larger number.

HCF × LCM = product of the two numbers
6 × 330 = 1980
Larger number = 1980 ÷ 30 = 66
Answer: 66

9 Cylinder of fixed surface area

A closed cylinder (a drum) has radius r cm, height h cm and total surface area 486π cm^2.
(a) Show that its volume, V cm^3, is $V = 243\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of π, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 486\pi$, so $h = \frac{243 - r^2}{r}$

$V = \pi r^2 h = \pi r(243 - r^2) = 243\pi r - \pi r^3$

$\frac{dV}{dr} = 243\pi - 3\pi r^2 = 0$, so $r^2 = 81$ and $r = 9$

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{243 - 81}{9} = 18 = 2r$

$V = \pi \times 9^2 \times 18 = 1458\pi$

Answer: 1458π cm^3

10 Equal roots and real roots

The equation $x^2 + kx + 16 = 0$ has exactly one solution.
Find the possible values of k.
Give any surds in their simplest form.

One solution means $b^2 - 4ac = 0$

$k^2 - 4 \times 1 \times 16 = 0$, so $k^2 = 64$

$k = \pm\sqrt{64} = \pm 8$

Answer: k = ±8