Day 47 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 18:40 (London time).
It takes 13 hours 30 minutes.
When it is 12:00 in London it is 17:00 in Delhi.
At what time, in Delhi time, does the flight land? Say if it is on a different day.

Arrival in London time = 18:40 + 13 h 30 min = 08:10 the next day
Delhi is 5 hours ahead of London, so add 5 hours
13:10 the next day
Answer: 13:10 the next day

2 Differentiating polynomials

$y = -3x^3 + 7x + 6$
Find $\frac{dy}{dx}$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$
$-3x^3 \to -9x^2$, $7x \to 7$
$\frac{dy}{dx} = -9x^2 + 7$
Answer: $\mathbf{\frac{dy}{dx} = -9x^2 + 7}$

3 Comparing costs

Sparkle Cleaners has a call-out fee of 50p plus £3.50 per hour.
Fresh Start has a call-out fee of £7.50 plus £3 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 18 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when 0.50 + 3.50n = 7.50 + 3n
0.50n = 7, so n = 14
(b) Sparkle Cleaners: 50p + £3.50 × 18 = £63.50
Fresh Start: £7.50 + £3 × 18 = £61.50
Fresh Start is cheaper by £2
Answer: (a) 14 (b) Fresh Start, by £2

4 Perpendicular bisector

A is the point (−3, −2) and B is the point (−5, −8).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−4, −5)
Gradient of AB = 3
Perpendicular gradient = $-\frac{1}{3}$
$y - (-5) = (-\frac{1}{3})(x - (-4))$
Rearrange to integer coefficients: x + 3y = −19
Answer: x + 3y = −19

5 Points with a given gradient (cubic)

The curve $y = 2x^3 + 9x^2 + 18x$ has gradient 6 at two points.
Find the x coordinates of these points.

$\frac{dy}{dx} = 6x^2 + 18x + 18$
$6x^2 + 18x + 18 = 6$, so $6x^2 + 18x + 12 = 0$
Divide by 6: $x^2 + 3x + 2 = 0$
(x + 2)(x + 1) = 0, so x = −2 or x = −1
Answer: x = −2 and x = −1

6 Finding an unknown from a matrix square

$A = \begin{pmatrix} 2 & k \\ -1 & 2 \end{pmatrix}$ and $A^2 = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$.

Find the value of k.

Top left entry of A^2: $2 \times 2 + k \times (-1) = 3$

$-k + 4 = 3$

$-k = -1$ so $k = 1$

Answer: k = 1

7 Rationalising the denominator

Write $\frac{1+\sqrt{7}}{3-\sqrt{7}}$ in the form $p + q\sqrt{7}$

where p and q are rational numbers.

Multiply top and bottom by $(3 + \sqrt{7})$:

Numerator: $(1 + \sqrt{7})(3 + \sqrt{7}) = 10 + 4\sqrt{7}$

Denominator: $9 - 7 = 2$

$= 5 + 2\sqrt{7}$

Answer: $5 + 2\sqrt{7}$

8 Stationary point of y = ax + b/√x

A curve has equation $y = 5x + \frac{10}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 5 - 5x^{-3/2}$

$\frac{dy}{dx} = 0$: $5x^{-3/2} = 5$, so $x^{3/2} = 1$

$x = 1^{2/3} = 1$, and $y = 15$

$\frac{d^2y}{dx^2} = \frac{15}{2}x^{-5/2}$

At $x = 1$, $\frac{d^2y}{dx^2} = \frac{15}{2}$, which is positive, so a minimum

Answer: (1, 15), minimum

9 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 6 : 6.
Tin B holds red paint and white paint in the ratio 4 : 1.
36 litres from tin A is mixed with some of tin B.
The ratio of red paint to white paint in the mixture is 7 : 4.
Work out how many litres were taken from tin B.

Let y litres be taken from B

red paint: $18 + \frac{4y}{5}$

white paint: $18 + \frac{1y}{5}$

Ratio 7 : 4, so $4(18 + \frac{4y}{5}) = 7(18 + \frac{1y}{5})$

$(\frac{9}{5})y = 54$, so $y = 30$

Answer: 30 litres

10 Bearings with sine and cosine rules

A ship sails 6 km from P to Q on a bearing of 042°.
It then sails 10 km from Q to R on a bearing of 163°.
(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 042° + 180° − 163° = 59°

$PR^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 59°$ so PR = 8.61 km

$\cos QPR = \frac{6^2 + 8.6137^2 - 10^2}{2 \times 6 \times 8.6137}$, angle QPR = 84.34°

Bearing of R from P = 042° + 84.34° = 126°

Answer: (a) 8.61 km (b) 126°