Day 46 **IGCSE 10-a-day: Model Solutions**

1 Estimating the median from grouped data

The table gives information about 98 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 9, 35, 55, 60, 88, 98
The median is the value at position 98 ÷ 2 = 49
It is in the class 40 to 60, with 35 values before it, so we need 14 more of the 20 values in this class

Median $\approx 40 + \frac{14}{20} \times 20 = 54$

Answer: 54

2 Two numbers: maximise or minimise

Two numbers x and y have a sum of 10.
Find the least possible value of $x^2 + y^2$, showing that it is the least.

$y = 10 - x$, so $x^2 + y^2 = x^2 + (10 - x)^2 = 2x^2 - 20x + 100$
$\frac{d}{dx}(x^2 + y^2) = 4x - 20 = 0$ gives $x = 5$
Second derivative = 4 > 0, so a minimum
x = y = 5, so the least value is 50
Answer: 50

3 Weighted percentages

The ratio of men to women in a school is 2 : 4.
85% of the men and 55% of the women own a car.
What percentage of all of these students own a car?

Imagine 120 students in total: 40 men and 80 women
85% of 40 = 34 and 55% of 80 = 44
Together: 78 out of 120
78 ÷ 120 × 100 = 65%
Answer: 65%

4 Expanding brackets with surds

Expand and simplify $(2 + 3\sqrt{3})(3 + \sqrt{3})$
Give your answer in the form $p + q\sqrt{3}$

$(2 + 3\sqrt{3})(3 + \sqrt{3}) = 6 + 2\sqrt{3} + 9\sqrt{3} + 9$
$= 15 + 11\sqrt{3}$
Answer: $15 + 11\sqrt{3}$

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 5 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 20 ÷ 5 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 5 \times 20 + 20 \times 15 + \frac{1}{2} \times 20 \times 20 = 50 + 300 + 200$
= 550
Answer: (a) 4 m/s^2 (b) 550 m

6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the hypotenuse AC, so use sin 45°

$\sin 45° = \frac{\sqrt{2}}{2}$

$x = 3 \times \sin 45° = 3 \times \frac{\sqrt{2}}{2}$

$x = \frac{3\sqrt{2}}{2}$

Answer: $x = \frac{3\sqrt{2}}{2}$

7 Fastest rate of growth

The number of visitors to a website, N, after t days is modelled by
$N = -2t^3 + 24t^2 + 12t + 200$ for $0 \leq t \leq 16$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -6t^2 + 48t + 12$

Rate is greatest when $\frac{d^2N}{dt^2} = -12t + 48 = 0$, so $t = 4$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -12), so the rate is a maximum

$\frac{dN}{dt} = 108$ when $t = 4$

Answer: t = 4, rate 108 per day

8 Probability with sets

$n(\xi) = 80$, $n(A) = 20$, $n(B) = 23$ and $n(A \cap B) = 18$.
Find $P(A \mid B)$ and $P(B \mid A')$.

$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{18}{23}$

$n(A') = 80 - 20 = 60$ and $n(B \cap A') = 23 - 18 = 5$

$P(B \mid A') = \frac{5}{60} = \frac{1}{12}$

Answer: $P(A \mid B) = \frac{18}{23}$, $P(B \mid A') = \frac{1}{12}$

9 Wire cut in two

A piece of wire 170 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is twice its width.

(a) Show that the total area, A cm², enclosed by the two shapes is $A = x^2 + \frac{(170 - 4x)^2}{18}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $170 - 4x = 6w$, so $w = \frac{170 - 4x}{6}$ and its area is $2w^2 = \frac{(170 - 4x)^2}{18}$

$A = x^2 + \frac{(170 - 4x)^2}{18} = \frac{17}{9}x^2 - \frac{680}{9}x + \frac{14450}{9}$

$\frac{dA}{dx} = \frac{34}{9}x - \frac{680}{9} = 0$, so $x = 20$. $\frac{d^2A}{dx^2} = \frac{34}{9} > 0$, so a minimum

Square: 400, rectangle: $30 \times 15 = 450$, so $A = 850$

Answer: x = 20, least area 850 cm²

10 Variation: percentage change

V is directly proportional to the square of r.
r is increased by 20%.
Work out the percentage change in V.

r is multiplied by 1.2

V is multiplied by 1.2^2

Multiplier for V = 1.44

Percentage increase = 44%

Answer: 44% increase