Day 45 **IGCSE 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the largest.

$\frac{14}{15}$, 0.61, 91%, 0.72

Convert each to a decimal: $\frac{14}{15}$ = 0.933, 0.61 = 0.61, 91% = 0.91, 0.72 = 0.72

In order: $\frac{14}{15}$, 91%, 0.72, 0.61

Answer: $\frac{14}{15}$, 91%, 0.72, 0.61

2 Expand then differentiate

$y = (x + 1)(x - 3)$

Find $\frac{dy}{dx}$.

Expand: $y = x^2 - 2x - 3$

$x^2 \rightarrow 2x$, $-2x \rightarrow -2$

$\frac{dy}{dx} = 2x - 2$

Answer: $\frac{dy}{dx} = 2x - 2$

3 Distance-time graphs

The graph shows Tariq's cycle ride to a café and back.

(a) Work out Tariq's speed on the way back.

(b) Work out Tariq's average speed for the whole ride, including the stop.

(a) Return: 45 km in 3 hours, so speed = 45 ÷ 3 = 15 km/h

(b) Total distance = 2 × 45 = 90 km, total time = 5 hours

Average speed = 90 ÷ 5 = 18 km/h

Answer: (a) 15 km/h (b) 18 km/h

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 43°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle APB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 43 = 86°

(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angle APB = 360 − 90 − 90 − 86 = 94°

Answer: (a) 86° (b) 94°

5 Largest domain and range

$f(x) = \frac{1}{\sqrt{x + 2}}$.

(a) State the largest possible domain of f.

(b) State the range of f for this domain.

(a) The square root must be positive (it is in the denominator): $x + 2 > 0$

Domain: $x > -2$

(b) 1 ÷ (a positive number) is positive, and can be as large as we like or as close to 0 as we like

Answer: (a) $x > -2$ (b) $f(x) > 0$

6 Trigonometric graphs

The diagram shows the graph of $y = a \sin x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 90$, $\sin x° = 1$, so $a + c = 9$
At $x = 270$, $\sin x° = -1$, so $-a + c = 3$
Add: $2c = 12$, so $c = 6$
Subtract: $2a = 6$, so $a = 3$
Answer: $a = 3$, $c = 6$

7 Tangent where the curve crosses the y-axis

The curve $y = 2x^2 - 2x + 4$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = 4$
$\frac{dy}{dx} = 4x - 2$, so the gradient at A is -2
Tangent: $y = -2x + 4$
At B, $y = 0$: $-2x + 4 = 0$, so $x = 2$
Answer: (2, 0)

8 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$6x + 3y \leq 60$ and $2x + 6y \leq 30$.
Find the greatest value of $P = 4x + 7y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 5), (9, 2), (10, 0)
Values of P: (0, 0): 0, (0, 5): 35, (9, 2): 50, (10, 0): 40
Greatest value of P is 50 at (9, 2)
Answer: $P = 50$ when $x = 9$ and $y = 2$

9 Ratio and algebra

In an animal shelter, the ratio of the number of cats to the number of dogs is 6 : 7.
Then 12 more cats are added and 6 more dogs are added.
The ratio is now 1 : 1.
How many dogs were there at the start?

Let the numbers at the start be $6k$ and $7k$
$(6k + 12) : (7k + 6) = 1 : 1$, so $1(6k + 12) = 1(7k + 6)$
$6k + 12 = 7k + 6$
$-1k = -6$, so $k = 6$
Number of dogs at the start: $7 \times 6 = 42$
Answer: 42

10 Increasing for all x

$f(x) = x^3 + kx^2 + 12x - 2$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 12$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 12 < 0$, so $4k^2 < 144$
$k^2 < 36$, so $-6 < k < 6$
Answer: $-6 < k < 6$