Day 44 **IGCSE 10-a-day: Model Solutions**

1 Sine rule: finding a side

Work out the length of AC.
Give your answer correct to 3 significant figures. (Calculator)

$\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$AC = \frac{11 \times \sin 64°}{\sin 80°}$

$= 10.0$

Answer: 10.0 cm

2 Laws of indices with algebra

Simplify fully $\frac{9x^2y \times 7x^5y^6}{3x^5y^6}$

Give your answer in the form kx^ay^b

Numbers: $9 \times 7 \div 3 = 21$
Powers of x: $2 + 5 - 5 = 2$
Powers of y: $1 + 6 - 6 = 1$
$= 21x^2y$

Answer: $21x^2y$

3 Compound interest

Priya invests £2,000 for 7 years at 3.5% per year compound interest.
Work out the value of the investment at the end of the 7 years.
Give your answer to the nearest penny. (Calculator)

Multiplier $= 1 + 3.5 \div 100 = 1.035$
$2000 \times 1.035^7 = 2544.56$
To the nearest penny: £2,544.56

Answer: £2,544.56

4 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 3**a**, OQ = 6**a** + 6**b** and OR = 10**a** + 14**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 3**a** + 6**b**
QR = OR − OQ = 4**a** + 8**b**

QR = $\frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4

Answer: (b) 3 : 4

5 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where

$s = 16t^{\frac{3}{2}} + 2t^2$ $(t \geq 0)$
Find the velocity and the acceleration of the particle when t = 9.

$v = \frac{ds}{dt} = 4t + 24t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = 4 + 12t^{-\frac{1}{2}}$

When t = 9: v = 108 and a = 8

Answer: v = 108 m/s, a = 8 m/s^2

6 Describe a transformation from a diagram

Triangle T is mapped onto triangle T′ by a single transformation.
(a) Describe fully this transformation.
(b) Write down the matrix that represents it.

(a) Compare corresponding vertices, e.g. $(-2, -3)$ goes to $(-2, -6)$
The transformation is a stretch, scale factor 2, x-axis invariant

(b) The columns show where (1, 0) and (0, 1) go: $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

Answer: (a) Stretch, scale factor 2, x-axis invariant (b) $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{pmatrix}$

7 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.
The point P(4, 3) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{3}{4}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{3}{4} = -\frac{4}{3}$

$y - 3 = (-\frac{4}{3})(x - 4)$

Multiply out and rearrange: $4x + 3y = 25$
(b) On the x-axis $y = 0$, so $4x = 25$
$x = \frac{25}{4}$, so T is $(\frac{25}{4}, 0)$

Answer: (a) 4x + 3y = 25 (b) T = $(\mathbf{\frac{25}{4}, 0})$

8 No stationary points

A curve has equation $y = 2x^3 + 18x^2 + 58x + 1$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 + 36x + 58 = 6(x + 3)^2 + 4$

A squared term is never negative, so $\frac{dy}{dx} \geq 4 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 4 (when $x = -3$)

Answer: (b) 4

9 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.
Over the two years the population increases by 10.25%.
Work out the value of x.

Yearly multiplier $= 1 + \frac{x}{100}$

Two years: $(1 + \frac{x}{100})^2 = 1.1025$

Take the square root: $1 + \frac{x}{100} = 1.05$

$\frac{x}{100} = 0.05$, so $x = 5$

Answer: x = 5

10 Normal to y = kx^2: show that

P is the point $(t, \frac{1}{2}t^2)$ on the curve $y = \frac{1}{2}x^2$, where $t \neq 0$.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{1}{2}t^2 + 1$.

$\frac{dy}{dx} = x$, so the gradient of the tangent at P is t

Normal gradient $= -\frac{1}{t}$

Normal: $y - \frac{1}{2}t^2 = -\frac{1}{t}(x - t)$

Put x = 0: $y = \frac{1}{2}t^2 + \frac{t}{t} = \frac{1}{2}t^2 + 1$

Answer: shown: $y = \frac{1}{2}t^2 + 1$