Day 43 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 19:50 (London time).
It takes 6 hours 15 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 19:50 + 6 h 15 min = 02:05 the next day
Cairo is 2 hours ahead of London, so add 2 hours
04:05 the next day
Answer: 04:05 the next day

2 Histogram with unequal class widths

The histogram shows the masses, in kg, of some parcels.
The bar for 20 to 40 represents 32 parcels.
(a) Work out the frequency of the class 0 to 20.
(b) Work out the total frequency.

Frequency = k × height × width. For 20 to 40: $32 = k \times 4 \times 20$ so $k = \frac{2}{5}$

(a) 0 to 20: $\frac{2}{5} \times 6 \times 20 = 48$

(b) Frequencies: 48, 32, 4, 12, 4
Total = 100
Answer: (a) 48 (b) 100

3 Fencing with a partition

A rectangular pen is x metres wide and y metres long. It is fenced all round and divided into two equal parts by a fence each of length x metres, parallel to the width.
The total length of fencing used is 36 m.

(a) Show that the area of the pen is $A = -\frac{3}{2}x^2 + 18x$.

(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $3x + 2y = 36$, so $y = \frac{36 - 3x}{2}$

$A = xy = \frac{x(36 - 3x)}{2} = -\frac{3}{2}x^2 + 18x$

$\frac{dA}{dx} = -3x + 18 = 0$ gives $x = 6$

$\frac{d^2A}{dx^2} = -3 < 0$, so a maximum

$A = 54$
Answer: 54 m²

4 Quadratic graphs

The curve $y = -x^2 + 4x - 3$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put $y = 0$. $-x^2 + 4x - 3 = 0$, so $-(x^2 + 4x + 3) = 0$, giving $-(x - 1)(x - 3) = 0$
$x - 1 = 0$ or $x - 3 = 0$, so $x = 1$ or $x = 3$
(b) The turning point is halfway between A and B: $x = (1 + 3) \div 2 = 2$
When $x = 2$: $y = -1 \times 2^2 + 4 \times 2 - 3 = 1$
Answer: (a) A(1, 0), B(3, 0) (b) (2, 1)

5 Tangent to a curve with negative or fractional powers

A curve has equation $y = -3x^2 - \frac{2}{x}$.

Find the equation of the tangent to the curve at the point where $x = 4$.

$\frac{dy}{dx} = -6x + 2x^{-2}$

At $x = 4$: $y = -\frac{97}{2}$ and gradient $= -\frac{191}{8}$

$y + \frac{97}{2} = -\frac{191}{8}(x - 4)$

$y = -\frac{191}{8}x + 47$

Answer: $y = -\frac{191}{8}x + 47$

6 Exact trigonometric ratios from a given ratio

x is an acute angle and $\cos x = \frac{3}{11}$.

(a) Find the exact value of sin x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - (\frac{3}{11})^2 = \frac{112}{121}$

x is acute, so sin x is positive: $\sin x = \frac{4\sqrt{7}}{11}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{4\sqrt{7}}{11} \div \frac{3}{11} = \frac{4\sqrt{7}}{3}$

Answer: (a) $\frac{4\sqrt{7}}{11}$ (b) $\frac{4\sqrt{7}}{3}$

7 Geometric sequences

The 3rd term of a geometric sequence is 108.

The 5th term of the sequence is 432.

All the terms of the sequence are positive.

(a) Work out the common ratio.

(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 432 \div 108 = 4$

$r = \sqrt{4} = 2$ (positive, since all terms are positive)

(b) The 3rd term is $a \times r^2$: $a \times 4 = 108$

$a = 108 \div 4 = 27$

Answer: (a) r = 2 (b) 27

8 Variation: joint and combined

y is directly proportional to x and inversely proportional to the square of z.

When x = 8 and z = 8, y = 0.625.

Find y when x = 10 and z = 2.

$y = k\frac{x}{z^2}$

Substitute: $0.625 = k \times \frac{1}{8}$ so $k = 5$

$y = 5 \times 2.5 = 12.5$

Answer: y = 12.5

9 Normal meets the curve again (quadratic)

The normal to the curve $y = -x^2 - 3x + 5$ at the point P where $x = -2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = -2x - 3$: at P, gradient = 1 and $y = 7$

Normal: $y = -x + 5$

Intersection: $-x^2 - 2x = 0$

One root is $x = -2$, and the sum of the roots is -2

So $x_Q = -2 - (-2) = 0$

$y = 5$

Answer: (0, 5)

10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.5.
If it rains, the probability that Ali is late for school is 0.3.
If it does not rain, the probability that Ali is late is 0.4.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.5 × 0.3 + 0.5 × 0.4 = 0.15 + 0.2 = 0.35

(b) P(rain and late) = 0.15

P(rain | late) = 0.15 ÷ 0.35 = $\frac{3}{7}$

Answer: (a) 0.35 (b) $\frac{3}{7}$