Day 42 **IGCSE 10-a-day: Model Solutions**

1 Parallel vectors

$\mathbf{a} = \begin{pmatrix} k \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

$\mathbf{a}$ is parallel to $\mathbf{b}$.

Find the value of k.

Parallel vectors are multiples of each other: $\mathbf{a} = \lambda\mathbf{b}$

$-4 = 2\lambda$ gives $\lambda = -2$

$k = -2 \times (-2) = 4$

Answer: k = 4

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = 2t^3 - 2t^2 + 3t + 3$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = 6t^2 - 4t + 3$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t - 4$

When t = 2: v = 19 and a = 20

Answer: (a) 19 m/s (b) 20 m/s^2

3 Iteration: rearranging equations

(a) Show that the equation $x^3 + 7x - 28 = 0$ can be rearranged to give $x = \sqrt[3]{28 - 7x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{28 - 7x_n}$ with $x_0 = 2$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 + 7x - 28 = 0$: $x^3 = 28 - 7x$

Take the cube root of both sides: $x = \sqrt[3]{28 - 7x}$

(b) $x_1 = \sqrt[3]{28 - 7 \times 2} = 2.41014...$

$x_2 = \sqrt[3]{28 - 7 \times x_1} = 2.23264...$

$x_1 = 2.410$ and $x_2 = 2.233$ (3 d.p.)

Answer: (b) $x_1 = 2.410$, $x_2 = 2.233$

4 Distance-time graphs

The graph shows Jamal's cycle ride to a café and back.

(a) Work out Jamal's speed on the way back.

(b) Work out Jamal's average speed for the whole ride, including the stop.

(a) Return: 60 km in 5 hours, so speed = 60 ÷ 5 = 12 km/h

(b) Total distance = 2 × 60 = 120 km, total time = 8 hours

Average speed = 120 ÷ 8 = 15 km/h

Answer: (a) 12 km/h (b) 15 km/h

5 Local maximum and minimum values

$f(x) = x^3 - 3x + 5$

Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$

$f'(x) = 0$ gives $x = -1$ or $x = 1$

$f''(x) = 6x$: $f''(-1) = -6 < 0$ (maximum) and $f''(1) = 6 > 0$ (minimum)

$f(-1) = 7$ and $f(1) = 3$

Answer: Local maximum y = 7, local minimum y = 3

6 Inequalities

(a) Solve $5 - 4x \geq -19$
(b) n is an integer. Write down all the values of n that satisfy
$-4 < 3n - 4 \leq 11$

(a) Subtract 5: $-4x \geq -24$
Divide by -4 and reverse the inequality sign: $x \leq 6$
(b) Add 4: $0 < 3n \leq 15$
Divide by 3: $0 < n \leq 5$
Integers: 1, 2, 3, 4, 5
Answer: (a) $x \leq 6$ (b) 1, 2, 3, 4, 5

7 Enlargement

Triangle P has vertices A(4, 4), B(10, 10) and C(2, 0).
Triangle Q has vertices A'(−2, 4), B'(−5, 1) and C'(−1, 6).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 6, x-step from A' to B' = −3

Scale factor = $-3 \div 6 = -\frac{1}{2}$

The centre C satisfies: $C + -\frac{1}{2}(A - C) = A'$, so $(1 - -\frac{1}{2})C = A' - -\frac{1}{2}A$

x: $\frac{3}{2} \times x = (-2) - (-\frac{1}{2}) \times 4 = 0$, so $x = 0$

y: $\frac{3}{2} \times y = 4 - (-\frac{1}{2}) \times 4 = 6$, so $y = 4$

Answer: Enlargement, scale factor $-\frac{1}{2}$, centre (0, 4)

8 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and $CB = \frac{2}{3}OA$.

$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.
(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.
(b) AM : MB = 1 : 2. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = \frac{2}{3}\mathbf{a}$

$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \frac{2}{3}\mathbf{a} + \mathbf{c}$

$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = -\frac{1}{3}\mathbf{a} + \mathbf{c}$

(b) $\mathbf{OM} = \mathbf{OA} + \frac{1}{3}\mathbf{AB} = \mathbf{a} + \frac{1}{3}(-\frac{1}{3}\mathbf{a} + \mathbf{c})$

$= \frac{8}{9}\mathbf{a} + \frac{1}{3}\mathbf{c}$

Answer: (a) $-\frac{1}{3}\mathbf{a} + \mathbf{c}$ (b) $\frac{8}{9}\mathbf{a} + \frac{1}{3}\mathbf{c}$

9 Counting factors

$N = 2^3 \times 3^5$
How many of the factors of N are odd numbers?

An odd factor contains no 2s
Ignore the 2 and count the factors of the rest: 6 = 6
Answer: 6

10 Normal meets the x-axis

The diagram shows the curve $y = 2x^2 - 2x + 4$ and the normal at P(1, 4), meeting the x-axis at Q. R is (1, 0).
(a) Find the normal in the form $x + ky = c$.
(b) Work out the area of triangle PQR.

$\frac{dy}{dx} = 4x - 2$, so the gradient of the tangent at P is 2

Normal gradient = $-\frac{1}{2}$

$y - 4 = -\frac{1}{2}(x - 1)$. Multiply by 2: $x + 2y = 9$

Q: $y = 0$ gives $x = 9$, so Q = (9, 0) and QR = 8
Area = ½ × 8 × 4 = 16
Answer: (a) x + 2y = 9 (b) 16