Day 41 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 16:50 (London time).
It takes 6 hours 30 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 16:50 + 6 h 30 min = 23:20
Tokyo is 8 hours ahead of London, so add 8 hours
07:20 the next day
Answer: 07:20 the next day

2 Vertical distance between stationary points

The curve $y = 2x^3 - 3x^2 - 12x + 4$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 - 6x - 12 = 6(x + 1)(x - 2)$
$x = -1$ and $x = 2$
At $x = -1$, $y = 11$; at $x = 2$, $y = -16$
Difference $= 11 - (-16) = 27$
Answer: 27

3 Inverse proportion in context

6 workers can build a wall in 18 days.
After 5 days, 3 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work $= 6 \times 18 = 108$ worker-days
Work done $= 6 \times 5 = 30$, so work left $= 78$
Workers left $= 6 - 3 = 3$
Time $= 78 \div 3 = 26$ days
Answer: 26 days

4 Largest domain and range

$f(x) = \sqrt{x + 6} + 2$.
(a) State the largest possible domain of f.
(b) State the range of f for this domain.

(a) The expression under the square root must be at least 0: $x + 6 \geq 0$
Domain: $x \geq -6$
(b) A square root is never negative, so the least value of f is 2
Answer: (a) $x \geq -6$ (b) $f(x) \geq 2$

5 Two numbers: maximise or minimise

Two numbers x and y have a sum of 18.
Find the least possible value of $x^2 + y^2$, showing that it is the least.

$y = 18 - x$, so $x^2 + y^2 = x^2 + (18 - x)^2 = 2x^2 - 36x + 324$
$\frac{d}{dx}(x^2 + y^2) = 4x - 36 = 0$ gives $x = 9$
Second derivative $= 4 > 0$, so a minimum
$x = y = 9$, so the least value is 162
Answer: 162

6 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - 3x + 6$

$y = -9x + 1$

Substitute: $x^2 - 3x + 6 = -9x + 1$

Rearrange: $x^2 + 6x + 5 = 0$

Factorise: $(x + 1)(x + 5) = 0$, so $x = -1$ or $x = -5$

When $x = -1$: $y = -9 \times (-1) + 1 = 10$

When $x = -5$: $y = -9 \times (-5) + 1 = 46$

Answer: $x = -5$, $y = 46$ and $x = -1$, $y = 10$

7 Equation of a transformed graph

$f(x) = x^3 + x^2 + x + 2$

The graph of $y = f(x)$ is transformed. Find the equation of the graph after it is

(a) stretched parallel to the y-axis with scale factor 3

(b) translated by the vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$

(a) $y = 3f(x)$: multiply every term by 3. So $y = 3x^3 + 3x^2 + 3x + 6$

(b) Replace x by $(x - 1)$ and add -5: $y = f(x - 1) - 5$. So $y = x^3 - 2x^2 + 2x - 4$

Answer: (a) $y = 3x^3 + 3x^2 + 3x + 6$ (b) $y = x^3 - 2x^2 + 2x - 4$

8 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = 2x^3 - 18x^2 + 48x - 6$ and the line $y = k$.

The line meets the curve at three different points.

Find the range of possible values of k.

$\frac{dy}{dx} = 6x^2 - 36x + 48 = 0$ gives $x = 2$ and $x = 4$

$y = 34$ at $x = 2$ and $y = 26$ at $x = 4$

The line crosses three times when it lies between the two turning-point values

$26 < k < 34$

Answer: $26 < k < 34$

9 Proof with indices

Show that $3^{n+2} - 3^n$ is a multiple of 8 for all integer values of n.

$3^{n+2} = 3^n \times 3^2$

$3^{n+2} - 3^n = 3^n(3^2 - 1)$

$3^2 - 1 = 8$

$= 8 \times 3^n$, which is a multiple of 8

Answer: 8×3^n

10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 3. N is on OB with ON : NB = 2 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{4}\mathbf{a} = \frac{1}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{2}{3}t\mathbf{b}$

Equate: $\frac{1}{4}s = 1 - t$ and $1 - s = \frac{2}{3}t$

Solving: $s = \frac{2}{5}$ and $t = \frac{9}{10}$

OX = $\frac{1}{10}\mathbf{a} + \frac{3}{5}\mathbf{b}$

Answer: (a) $\frac{1}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{10}\mathbf{a} + \frac{3}{5}\mathbf{b}$