Day 40 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 30 is taken. Work out how many come from Year 7.
(b) A different stratified sample has 12 from Year 8. Work out the size of this sample.

Total = 200 + 100 + 225 + 225 = 750

(a) Year 7: $\frac{200}{750} \times 30 = 8$

(b) Fraction sampled = $12 \div 100 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 750 = 90$

Answer: (a) 8 (b) 90

2 Increasing or decreasing at a point

$y = -x^3 + 12x + 3$
At the point where $x = -1$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = -3x^2 + 12$

When $x = -1$: $\frac{dy}{dx} = 9$

9 is positive, so the curve is increasing
Answer: increasing (gradient 9)

3 Line and curve intersections

Solve the simultaneous equations
$y = x^2 - 2x + 2$
$y = -2x + 18$

Substitute: $x^2 - 2x + 2 = -2x + 18$
Rearrange: $x^2 - 16 = 0$
Factorise: $(x - 4)(x + 4) = 0$, so $x = 4$ or $x = -4$
When $x = 4$: $y = -2 \times 4 + 18 = 10$
When $x = -4$: $y = -2 \times (-4) + 18 = 26$
Answer: $x = -4$, $y = 26$ and $x = 4$, $y = 10$

4 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where
$s = 8t^{\frac{3}{2}} + t^2$ $(t \geq 0)$
Find the velocity and the acceleration of the particle when $t = 16$.

$v = \frac{ds}{dt} = 2t + 12t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = 2 + 6t^{-\frac{1}{2}}$

When $t = 16$: $v = 80$ and $a = \frac{7}{2}$

Answer: $v = 80$ m/s, $a = \frac{7}{2}$ m/s^2

5 Recurring decimals

Show that $0.\dot{5} = \frac{5}{9}$

Let $x = 0.5555...$
$10x = 5.5555...$
Subtract: $9x = 5$
$x = \frac{5}{9}$

Answer: $x = \frac{5}{9}$

6 Tree diagrams and reverse conditional probability

The probability that it rains is $\frac{3}{4}$.

If it rains, the probability that Sam is late is $\frac{1}{6}$. If it does not rain, the probability is $\frac{2}{3}$.

(a) Work out the probability that Sam is late.

(b) Given that Sam is late, work out the probability that it rained.

(a) P(rain and late) = $\frac{3}{4} \times \frac{1}{6} = \frac{1}{8}$; P(no rain and late) = $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$

P(late) = $\frac{1}{8} + \frac{1}{6} = \frac{7}{24}$

(b) P(rain | late) = $\frac{\frac{1}{8}}{\frac{7}{24}} = \frac{3}{7}$

Answer: (a) $\frac{7}{24}$ (b) $\frac{3}{7}$

7 Similar triangles with algebra

In the diagram, DE is parallel to BC.

AD = x cm, DB = (x + 6) cm, DE = 3 cm and BC = 7 cm.

(a) Find the value of x.

(b) Work out the length of AB.

The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 6} = \frac{3}{7}$

$7x = 3(2x + 6)$

$7x = 6x + 18$

$x = 18$

(b) AB = 2x + 6 = 42 cm

Answer: (a) x = 18 (b) 42 cm

8 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by

$N = -2t^3 + 18t^2 + 10t + 200$ for $0 \leq t \leq 12$

Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -6t^2 + 36t + 10$

Rate is greatest when $\frac{d^2N}{dt^2} = -12t + 36 = 0$, so t = 3

$\frac{d^2N}{dt^2}$ is decreasing (gradient −12), so the rate is a maximum

$\frac{dN}{dt} = 64$ when t = 3

Answer: t = 3, rate 64 per hour

9 Rationalising the denominator

Write $\frac{3 + \sqrt{7}}{3 - \sqrt{7}}$ in the form $p + q\sqrt{7}$

where p and q are rational numbers.

Multiply top and bottom by $(3 + \sqrt{7})$:

Numerator: $(3 + \sqrt{7})(3 + \sqrt{7}) = 16 + 6\sqrt{7}$

Denominator: 9 − 7 = 2

$= 8 + 3\sqrt{7}$

Answer: $8 + 3\sqrt{7}$

10 Column vector arithmetic

$\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$.

(a) Work out $\mathbf{a} + \mathbf{b}$.

(b) Work out $4\mathbf{a} - 3\mathbf{b}$.

(a) $\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$

(b) $4\mathbf{a} = \begin{pmatrix} 20 \\ -8 \end{pmatrix}$ and $3\mathbf{b} = \begin{pmatrix} -9 \\ -12 \end{pmatrix}$

$\begin{pmatrix} 20 \\ -8 \end{pmatrix} - \begin{pmatrix} -9 \\ -12 \end{pmatrix} = \begin{pmatrix} 29 \\ 4 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{2} \\ \mathbf{-6} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{29} \\ \mathbf{4} \end{pmatrix}$