Day 4 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 08:15 (London time).
It takes 9 hours 15 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 08:15 + 9 h 15 min = 17:30
Tokyo is 8 hours ahead of London, so add 8 hours
01:30 the next day
Answer: 01:30 the next day

2 Multiplying 2 by 2 matrices

A = $\begin{pmatrix} 1 & -2 \\ 2 & -2 \end{pmatrix}$ and B = $\begin{pmatrix} 5 & 0 \\ 5 & 3 \end{pmatrix}$.
(a) Work out AB.
(b) Work out BA.

(a) AB: row 1 is $1 \times 5 + (-2) \times 5$ and $1 \times 0 + (-2) \times 3$; row 2 is $2 \times 5 + (-2) \times 5$ and $2 \times 0 + (-2) \times 3$
AB = $\begin{pmatrix} -5 & -6 \\ 0 & -6 \end{pmatrix}$
(b) BA = $\begin{pmatrix} 5 & -10 \\ 11 & -16 \end{pmatrix}$
Answer: (a) AB = $\begin{pmatrix} \mathbf{-5} & \mathbf{-6} \\ \mathbf{0} & \mathbf{-6} \end{pmatrix}$ **(b) BA** = $\begin{pmatrix} \mathbf{5} & \mathbf{-10} \\ \mathbf{11} & \mathbf{-16} \end{pmatrix}$

3 f′(x) notation

$f(x) = 3x^2 + 6x - 4$
(a) Find f′(x).
(b) Work out the value of f′(2).

(a) $3x^2 \to 6x$, $6x \to 6$
f′(x) = 6x + 6
(b) f′(2) = 18
Answer: (a) f′(x) = 6x + 6 (b) 18

4 Solve f(x) = g(x)

$f(x) = 3x^2 + 5x - 6$ and $g(x) = -10x - 18$.
Solve f(x) = g(x).

$3x^2 + 5x - 6 = -10x - 18$
$3x^2 + 15x + 12 = 0$
$3(x + 4)(x + 1) = 0$
$x = -4$ or $x = -1$
Answer: x = −4 or x = −1

5 Stationary point of a quartic

$y = -x^4 - 32x$
Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -4x^3 - 32$
$\frac{dy}{dx} = 0$: $-4x^3 - 32 = 0$, so $x^3 = -8$
$x = -2$ and $y = 48$
$\frac{d^2y}{dx^2} = -12x^2$
At $x = -2$: $\frac{d^2y}{dx^2} = -48$, so a maximum
Answer: (−2, 48), maximum

6 Probabilities with an unknown

A biased wheel stops on one of four outcomes with these probabilities.
A: 2x; B: 2x + 0.2; C: 2x + 0.2; D: 0.3
(a) Work out the value of x.
(b) The biased wheel is used 40 times. Work out an estimate for the number of times it stops on A.

(a) Probabilities add up to 1: 2x + 2x + 0.2 + 2x + 0.2 + 0.3 = 1
6x + 0.7 = 1, so 6x = 0.3
x = 0.05
(b) P(A) = 2 × 0.05 = 0.1, so 40 × 0.1 = 4
Answer: (a) x = 0.05 (b) 4

7 Minimum distance with 1/t

A particle P moves on a straight line. At time t seconds (t > 0), its distance from a point O is s metres, where
$s = 3t^2 + \frac{6}{t}$
Show that P is at its minimum distance from O when t = 1, and find this minimum distance.

$s = 3t^2 + 6t^{-1}$, so $v = \frac{ds}{dt} = 6t - 6t^{-2}$

v = 0: $6t = \frac{6}{t^2}$, so $t^3 = 1$ and t = 1

$\frac{d^2s}{dt^2} = 6 + \frac{12}{t^3}$, which is 18 when t = 1, so a minimum
s = 9
Answer: 9 m

8 Cone made from a sector

A sector of a circle has radius 30 cm and angle 240°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{240}{360} \times 2\pi \times 30 = 40\pi$ cm
This is the circumference of the base: $2\pi r = 40\pi$, so r = 20 cm
(b) Slant height = 30 cm, so height$^2 = 30^2 - 20^2 = 500$, height = $10\sqrt{5}$ cm
Volume = $\frac{1}{3} \times \pi \times 20^2 \times 10\sqrt{5} = \frac{4000\sqrt{5}}{3}\pi$ cm^3
Answer: (a) 20 cm (b) $\frac{4000\sqrt{5}}{3}\pi$ cm^3

9 Profit and loss

A retailer buys a guitar and adds a 40% mark-up to get the selling price.
In a sale the selling price is reduced by 20% to £436.80.
Work out how much the shop paid for a guitar.

Overall multiplier = 1.4 × 0.8 = 1.12
Cost = £436.80 ÷ 1.12
= £390
Answer: £390

10 Tangency conditions

The line $y = kx - 6$ is a tangent to the curve $y = x^2 - 3$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 - 3 = kx - 6$

Rearrange: $x^2 - kx + 3 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 3 = 0$

$k^2 = 12$

$k = \pm\sqrt{12} = \pm 2\sqrt{3}$

Answer: $k = \pm 2\sqrt{3}$