Day 39 **IGCSE 10-a-day: Model Solutions**

1 Given a stationary point, find k

The curve $y = x^3 + 3x^2 + kx$ has a stationary point at $x = 1$.
(a) Find the value of k.
(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + 6x + k$

$x = 1$: $9 + k = 0$, so $k = -9$

$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x - 1)(x + 3)$

Other root: $x = -3$

$y = 27$

Answer: (a) $k = -9$ (b) $(-3, 27)$

2 Median and quartiles from a list

Here are 7 numbers.
11, 21, 9, 54, 35, 54, 4
Find (a) the median (b) the interquartile range.

Ordered: 4, 9, 11, 21, 35, 54, 54
(a) Median is the 4th value = 21
(b) Lower quartile is the 2nd value = 9; upper quartile is the 6th value = 54
IQR = 54 − 9 = 45
Answer: (a) 21 (b) 45

3 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 10 ÷ 10 = 1 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle

$\frac{1}{2} \times 10 \times 10 + 10 \times 8 + \frac{1}{2} \times 5 \times 10 = 50 + 80 + 25$

$= 155$

Answer: (a) 1 m/s^2 (b) 155 m

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(3x + 15)°$, angle BCD = $(5x - 11)°$ and angle ABC = $(2x + 15)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(3x + 15) + (5x - 11) = 180$
$8x + 4 = 180$, so $8x = 176$
$x = 22$
(b) Angle ABC = $2 \times 22 + 15 = 59°$
Angle ADC = $180 - 59 = 121°$
Answer: (a) $x = 22$ (b) 121°

5 Product of brackets: tangent gradient

$y = x(x + 4)(3x - 2)$
Work out the gradient of the curve at the point where $x = -2$.

Expand: $y = 3x^3 + 10x^2 - 8x$

$\frac{dy}{dx} = 9x^2 + 20x - 8$

$x = -2$: $\frac{dy}{dx} = -12$

Answer: -12

6 Three picks without replacement

A bag has 4 black and 5 red cubes.
Three cubes are taken at random without replacement.
(a) Work out the probability that all three are red.
(b) Work out the probability that at least one is black.

(a) $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

(b) At least one black is the opposite of all three red

$1 - \frac{5}{42} = \frac{37}{42}$

Answer: (a) $\frac{5}{42}$ (b) $\frac{37}{42}$

7 Tree diagrams without replacement

A bag contains 4 red counters and 3 blue counters.
Two counters are taken at random without replacement.
Work out the probability that (a) exactly one is red, (b) at least one is red.

(a) P(exactly one red) = $\frac{4}{7}\times\frac{3}{6} + \frac{3}{7}\times\frac{4}{6} = \frac{4}{7}$

(b) P(at least one red) = 1 − P(none) = $1 - \frac{3}{7}\times\frac{2}{6} = \frac{6}{7}$

Answer: (a) $\frac{4}{7}$ (b) $\frac{6}{7}$

8 Normal to y = kx^2: show that

P is the point $(t, 3t^2)$ on the curve $y = 3x^2$, where $t \neq 0$.
The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $3t^2 + \frac{1}{6}$.

$\frac{dy}{dx} = 6x$, so the gradient of the tangent at P is 6t

Normal gradient = $-\frac{1}{6t}$

Normal: $y - 3t^2 = -\frac{1}{6t}(x - t)$

Put x = 0: $y = 3t^2 + \frac{t}{6t} = 3t^2 + \frac{1}{6}$

Answer: shown: $y = 3t^2 + \frac{1}{6}$

9 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 5^2 + 10^2 - 2 \times 5 \times 10 \times \cos 40°$
$x^2 = 48.396$
x = 7.0 cm (1 d.p.)

(b) Area = $\frac{1}{2} \times 5 \times 10 \times \sin 40° = 16.1$ cm^2 (3 s.f.)

Answer: (a) 7.0 cm (b) 16.1 cm^2

10 Percentages with an unknown rate

The price of a car is increased by x%.
The new price is then decreased by x%.
The final price is 1% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 1% less, so the multiplier is 0.99: $1 - \frac{x^2}{10000} = 0.99$

$\frac{x^2}{10000} = 0.01$, so $x^2 = 100$ and $x = 10$

Answer: x = 10