Day 38 **IGCSE 10-a-day: Model Solutions**

1 Distance-time graphs

The graph shows Amir's cycle ride to a café and back.
(a) Work out Amir's speed on the way back.
(b) Work out Amir's average speed for the whole ride, including the stop.

(a) Return: 30 km in 2 hours, so speed = 30 ÷ 2 = 15 km/h
(b) Total distance = 2 × 30 = 60 km, total time = 5 hours
Average speed = 60 ÷ 5 = 12 km/h
Answer: (a) 15 km/h (b) 12 km/h

2 Surds with unknowns

Given that $(a + \sqrt{15})^2 = 64 + 14\sqrt{15}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{15})^2 = a^2 + 2a\sqrt{15} + 15$
Compare the $\sqrt{15}$ terms: $2a = 14$, so $a = 7$
Check the integer terms: $a^2 + 15 = 49 + 15 = 64$
Answer: a = 7

3 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by
$h = -2t^2 + 12t + 2$
(a) Work out the greatest height reached by the ball.
(b) Work out the velocity of the ball when t = 9. State whether it is moving up or down.

$\frac{dh}{dt} = -4t + 12 = 0$ gives $t = 3$
$\frac{d^2h}{dt^2} = -4 < 0$ so maximum. h = 20
When t = 9: $\frac{dh}{dt} = 12 - 4 \times 9 = -24$
Answer: (a) 20 m (b) −24 m/s, moving down

4 Composite functions

$f(x) = 4x - 1$ and $g(x) = 5x^2 - 8$
(a) Find fg(x).
(b) Solve fg(x) = 467

(a) $fg(x) = f(5x^2 - 8) = 4(5x^2 - 8) - 1$
$= 20x^2 - 33$
(b) $20x^2 - 33 = 467$, so $20x^2 = 500$
$x^2 = 25$, so $x = \pm 5$
Answer: (a) $20x^2 - 33$ (b) $x = \pm 5$

5 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 12^2 + 9^2 = 225$
$AG^2 = AC^2 + CG^2 = 225 + 8^2 = 289$
AG = 17 cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{8}{\sqrt{225}}$
angle GAC = 28.1° (1 d.p.)
Answer: (a) 17 cm (b) 28.1°

6 Stationary points of a quartic

A curve has equation $y = x^4 - 8x^2 + 7$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 16x = 4x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 2$ or $x = -2$

$y = 7$ when $x = 0$, and $y = -9$ when $x = \pm 2$

$\frac{d^2y}{dx^2} = 12x^2 - 16$: it is -16 at $x = 0$ (maximum) and 32 at $x = \pm 2$ (minimum)

Answer: (0, 7) maximum; (2, −9) and (−2, −9) minima

7 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 4**a** + **b**, OQ = 7**a** + 4**b** and OR = 8**a** + 5**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 3**a** + 3**b**
QR = OR − OQ = **a** + **b**

QR = $\frac{1}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 1
Answer: (b) 3 : 1

8 Venn diagram with three sets: algebra

The Venn diagram shows information about 34 shoppers.
A: bought fruit B: bought bread C: bought milk
(a) Find the value of x.
(b) Find n(A) and n(B ∪ C)′.

(a) 3x + 25 = 34
3x = 9, so x = 3
(b) Substitute x = 3 to fill in every region
n(A) = 4 + 4 + 3 + 7 = 18
n(B ∪ C)′ = 7 + 7 = 14 (A only plus outside)
Answer: (a) x = 3 (b) n(A) = 18, n(B ∪ C)′ = 14

9 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + 3x + 6$ at the point P where $x = -2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 3$: at P, gradient = −1 and y = 4

Normal: $y = x + 6$

Intersection: $x^2 + 2x = 0$
One root is $x = -2$, and the sum of the roots is -2
So $x_Q = -2 - (-2) = 0$
$y = 6$
Answer: (0, 6)

10 Variation in context

The mass M kg of a sphere of a metal is directly proportional to the cube of its radius r cm.
When r = 6, M = 324 kg.
Find the value of M when r = 5.

$M = kr^3$

$324 = k \times 6^3$ so $k = \frac{3}{2}$

$M = \frac{3}{2}r^3$

$M = \frac{3}{2} \times 125 = 187.5$

Answer: 187.5 kg