Day 37 **IGCSE 10-a-day: Model Solutions**

1 Inequalities

(a) Solve $4 - 5x \leq -1$
(b) n is an integer. Write down all the values of n that satisfy
$-15 < 3n - 3 < 6$

(a) Subtract 4: $-5x \leq -5$
Divide by -5 and reverse the inequality sign: $x \geq 1$
(b) Add 3: $-12 < 3n < 9$
Divide by 3: $-4 < n < 3$
Integers: $-3, -2, -1, 0, 1, 2$
Answer: (a) $x \geq 1$ (b) $-3, -2, -1, 0, 1, 2$

2 Gradient with fractional or negative powers

The equation of a curve is $y = -3x^2 + \frac{4}{x}$.

Work out the gradient of the curve at the point where $x = -2$.

$\frac{dy}{dx} = -6x - 4x^{-2}$
When $x = -2$: gradient = 11
Answer: 11

3 Forming and solving quadratic equations

The sum of the squares of two consecutive positive integers is 41.
Let the smaller integer be n.
(a) Show that $n^2 + n - 20 = 0$
(b) Work out the two integers.

(a) $n^2 + (n + 1)^2 = 41$, so $2n^2 + 2n + 1 = 41$
$2n^2 + 2n = 40$, so $n^2 + n - 20 = 0$
(b) Factorise: $(n - 4)(n + 5) = 0$
$n = 4$ or $n = -5$. n is positive, so $n = 4$
The integers are 4 and 5
Answer: (b) 4 and 5

4 Stationary points of y = ax + b/x

$y = x + \frac{4}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 1 - \frac{4}{x^2}$
$\frac{dy}{dx} = 0$: $\frac{4}{x^2} = 1$, so $x^2 = 4$ and $x = 2$ or $x = -2$
$y = 4$ when $x = 2$ and $y = -4$ when $x = -2$
$\frac{d^2y}{dx^2} = \frac{8}{x^3}$
$x = 2$: $\frac{d^2y}{dx^2} = 1$ so minimum; $x = -2$: $\frac{d^2y}{dx^2} = -1$ so maximum
Answer: (2, 4) minimum; (−2, −4) maximum

5 Percentages of percentages

In a group of students, 60% are boys and the rest are girls.
45% of the boys and 25% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 60 are boys and 40 are girls
45% of 60 = 27 and 25% of 40 = 10
27 + 10 = 37 out of 100
Answer: 37%

6 Equation of a normal

A curve has equation $y = 3x^2 - 3x + 5$.
Find the equation of the normal to the curve at the point where $x = 0$.

When $x = 0$, $y = 5$ and $\frac{dy}{dx} = -3$

Gradient of the normal $= -1 \div (-3) = \frac{1}{3}$

$y - 5 = \frac{1}{3}x$

$y = \frac{1}{3}x + 5$

Answer: $y = \frac{1}{3}x + 5$

7 Venn diagram with algebra

The Venn diagram shows information about 44 pupils.
A: walk to school B: cycle to school
(a) Find the value of x.
(b) Find n(A) and n(B ∩ A′).

(a) $5x + 14 = 44$
$5x = 30$, so $x = 6$
(b) $n(A) = x + 3 + x = 9 + 6 = 15$
$n(B \cap A') = 3x + 2 = 20$
Answer: (a) x = 6 (b) n(A) = 15, n(B ∩ A′) = 20

8 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 10 cm and inner radius 9.5 cm. (Calculator)
The density of the metal is 7.8 g/cm^3.
The mass of the pipe is 7.17 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 7170 g, so volume = mass ÷ density = 7170 ÷ 7.8 = 919.23 cm^3
Cross-sectional area = $\pi \times 10^2 - \pi \times 9.5^2 = 30.631$ cm^2
Length = volume ÷ area = 919.23 ÷ 30.631 = 30.0 cm (3 s.f.)
Answer: 30.0 cm

9 Bounds: finding an unknown length

A rectangle has area 39 cm^2, correct to 2 significant figures.
Its length is 8.800000000000000710542735760100185871124267578125 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $38.5 \leq A < 39.5$. Length: $8.750000000000000710542735760100185871124267578125 \leq l < 8.850000000000000710542735760100185871124267578125$
UB width = UB area ÷ LB length = 39.5 ÷ 8.750000000000000710542735760100185871124267578125 = 4.51
Answer: 4.51 cm

10 Probability and quadratic equations

A bag contains some sweets. 3 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{3}{28}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

P(both red) = $\frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

$\frac{6}{n(n-1)} = \frac{3}{28}$, so $n(n-1) = 56$, giving $n^2 - n - 56 = 0$

$(n-8)(n+7) = 0$, so $n = 8$ (n cannot be negative)

Number of sweets that are yellow = $8 - 3 = 5$

Answer: 5