Day 365 **IGCSE 10-a-day: Model Solutions**

1 Points satisfying inequalities

R is the region that satisfies all of these inequalities:
$y < 3x - 2$ $x + y \geq 9$ $x > 1$
Which of these points lie in R?
A(1, 8) B(1, 10) C(2, 2) D(4, 3) E(7, 7)

A(1, 8): $y < 1$: no; $x + y = 9$: yes; $x = 1$: no
B(1, 10): $y < 1$: no; $x + y = 11$: yes; $x = 1$: no
C(2, 2): $y < 4$: yes; $x + y = 4$: no; $x = 2$: yes
D(4, 3): $y < 10$: yes; $x + y = 7$: no; $x = 4$: yes
E(7, 7): $y < 19$: yes; $x + y = 14$: yes; $x = 7$: yes
In R: E
Answer: E

2 Factorising fully

Factorise fully $(x + 6)^2 - 4$

This is a difference of two squares: $(x + 6)^2 - 2^2$
$= (x + 6 - 2)(x + 6 + 2)$
$= (x + 4)(x + 8)$
Answer: $(x + 4)(x + 8)$

3 Compound interest

Evan invests £1,500.
The account pays compound interest of 4% a year for the first 2 years,
then 1.5% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.04 for 2 years, then 1.015 for 2 years
$£1{,}500 \times 1.04^2 \times 1.015^2$
= £1,671.44
Answer: £1,671.44

4 Stationary points of y = ax + b/x

$y = 2x + \frac{32}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 2 - \frac{32}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{32}{x^2} = 2$, so $x^2 = 16$ and $x = 4$ or $x = -4$

$y = 16$ when $x = 4$ and $y = -16$ when $x = -4$

$\frac{d^2y}{dx^2} = \frac{64}{x^3}$

$x = 4$: $\frac{d^2y}{dx^2} = 1$ so minimum; $x = -4$: $\frac{d^2y}{dx^2} = -1$ so maximum

Answer: (4, 16) minimum; (−4, −16) maximum

5 Area of a triangle using sine

Triangle ABC has AB = 18 cm and BC = 10 cm.
The area of triangle ABC is $45\sqrt{3}$ cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$45\sqrt{3} = \frac{1}{2} \times 18 \times 10 \times \sin B$

$\sin B = 45\sqrt{3} \div 90 = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$, and B is acute, so B = 60°

Answer: 60°

6 Reverse percentages

The price of a bike is increased by 50%.
Later the new price is reduced by 25% in a sale.
The sale price is £247.50.
Work out the price before the increase. (Calculator)

Multipliers: 1.5 and 0.75
Overall multiplier = 1.5 × 0.75 = 1.125
Original × 1.125 = £247.50
Original = £247.50 ÷ 1.125 = £220
Answer: £220

7 Tangent parallel to a chord

The diagram shows part of the curve $y = -x^2 + 2x + 6$. A and B are the points on the curve with x coordinates −1 and 1.
C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = (−1, 3) and B = (1, 7)

Gradient of AB = $\frac{7 - 3}{1 - (-1)} = 2$

$\frac{dy}{dx} = -2x + 2 = 2$, so x = 0

y = 6
Answer: C (0, 6)

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 2**a** + **b**, OQ = −7**a** − 11**b** and OR = −13**a** − 19**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = −9**a** − 12**b**
QR = OR − OQ = −6**a** − 8**b**

QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

9 Linear programming in context

A craft shop makes x necklaces and y bracelets each day.
Each necklace needs 1 minute of making time and 4 beads. Each bracelet needs 3 minutes of making time and 4 beads.
There are 21 minutes of making time and 44 beads available.
The profit is £3 for each necklace and £6 for each bracelet.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $x + 3y \leq 21$ and $4x + 4y \leq 44$
(b) Vertices of the feasible region: (0, 0), (0, 7), (6, 5), (11, 0)
Profit = 3x + 6y: 0, 42, 48, 33
Greatest profit = £48 with x = 6, y = 5
Answer: (a) $x + 3y \leq 21$, $4x + 4y \leq 44$ (b) £48

10 Normal meets the curve again (quadratic)

The normal to the curve $y = -x^2 + 4x + 6$ at the point P where x = 1 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = -2x + 4$: at P, gradient = 2 and y = 9

Normal: $y = -\frac{1}{2}x + \frac{19}{2}$

Intersection: $-x^2 + \frac{9}{2}x - \frac{7}{2} = 0$

One root is x = 1, and the sum of the roots is $\frac{9}{2}$

So $x_Q = \frac{9}{2} - 1 = \frac{7}{2}$

$y = \frac{31}{4}$

Answer: $\left(\frac{7}{2}, \frac{31}{4}\right)$