Day 364 **IGCSE 10-a-day: Model Solutions**

1 Find unknown coefficients from gradients

The curve $y = x^3 + ax^2 + bx$ has gradient 21 when $x = 1$ and gradient 8 when $x = 0$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = 1$: $2a + b = 18$
$x = 0$: $b = 8$
Subtract and solve: $a = 5$
$b = 8$
Answer: a = 5, b = 8

2 Forming and solving quadratic equations

A rectangle has length $(2x + 6)$ cm and width $(x + 3)$ cm.
The area of the rectangle is 162 cm^2.
(a) Show that $2x^2 + 12x - 144 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(2x + 6)(x + 3) = 162$, so $2x^2 + 12x + 18 = 162$
$2x^2 + 12x - 144 = 0$
(b) $2x^2 + 12x - 144 = 0$: $2(x - 6)(x + 12) = 0$
$x = 6$ ($x = -12$ is not possible as lengths are positive)
Length = 18 cm and width = 9 cm, so perimeter = 2 × (18 + 9) = 54 cm
Answer: (b) 54 cm

3 Distance-time graphs

The graph shows Tariq's cycle ride to a café and back.
(a) Work out Tariq's speed on the way back.
(b) Work out Tariq's average speed for the whole ride, including the stop.

(a) Return: 15 km in 1 hours, so speed = 15 ÷ 1 = 15 km/h
(b) Total distance = 2 × 15 = 30 km, total time = 2 hours
Average speed = 30 ÷ 2 = 15 km/h
Answer: (a) 15 km/h (b) 15 km/h

4 Increasing and decreasing intervals

$y = -x^3 + 9x^2 - 24x - 5$
Find the range of values of x for which y is increasing.

$\frac{dy}{dx} = -3x^2 + 18x - 24 = -3(x - 2)(x - 4)$
$\frac{dy}{dx} = 0$ when $x = 2$ and $x = 4$
The graph of $\frac{dy}{dx}$ against x is a ∩-shaped quadratic
y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$
So y is increasing when $2 < x < 4$
Answer: $2 < x < 4$

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 4 cm and the area of triangle ADE is 4 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 4 = 8 cm
Length scale factor = 8 ÷ 4 = 2
Area scale factor = $(2)^2 = 4$
Area of triangle ABC = 4 × 4 = 16 cm^2
Trapezium = 16 − 4 = 12 cm^2
Answer: 12 cm^2

6 Exact lengths and areas

A right-angled triangle has base $(6 + \sqrt{3})$ cm and height $(6 - \sqrt{3})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area = $\frac{1}{2} \times (6 + \sqrt{3})(6 - \sqrt{3})$
$(6 + \sqrt{3})(6 - \sqrt{3}) = 36 - 3 = 33$
Area = $\frac{1}{2} \times 33 = \frac{33}{2}$
Answer: $\frac{33}{2}$ cm^2

7 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (2, 8).
The curve crosses the y-axis at (0, 4).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (2, 8), so $y = a(x - 2)^2 + 8$
When x = 0, y = 4: $a \times 4 + 8 = 4$, so $a = -1$
$y = -(x - 2)^2 + 8$
Expand: $y = -x^2 + 4x + 4$
Answer: $y = -x^2 + 4x + 4$

8 Tree diagrams and reverse conditional probability

The probability that it rains is $\frac{4}{5}$.

If it rains, the probability that Sam is late is $\frac{1}{2}$. If it does not rain, the probability is $\frac{5}{6}$.

(a) Work out the probability that Sam is late.
(b) Given that Sam is late, work out the probability that it rained.

(a) P(rain and late) = $\frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$; P(no rain and late) = $\frac{1}{5} \times \frac{5}{6} = \frac{1}{6}$
P(late) = $\frac{2}{5} + \frac{1}{6} = \frac{17}{30}$
(b) P(rain | late) = $\frac{\frac{2}{5}}{\frac{17}{30}} = \frac{12}{17}$
Answer: (a) $\frac{17}{30}$ (b) $\frac{12}{17}$

9 Tangent parallel to a chord

The diagram shows part of the curve $y = -2x^2 - 4x + 5$. A and B are the points on the curve with x coordinates 0 and 4. C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = (0, 5) and B = (4, −43)

Gradient of AB = $\frac{-43 - 5}{4 - 0} = -12$

$\frac{dy}{dx} = -4x - 4 = -12$, so x = 2

y = −11

Answer: C (2, −11)

10 Variation: percentage change

P is directly proportional to the square of t.
t is decreased by 50%.
Work out the percentage change in P.

t is multiplied by 0.5

P is multiplied by 0.5^2

Multiplier for P = 0.25

Percentage decrease = 75%

Answer: 75% decrease