Day 363 **IGCSE 10-a-day: Model Solutions**

1 Inverse of a linear function

$f(x) = \frac{x - 2}{6}$.

(a) Find $f^{-1}(x)$.

(b) Find $f^{-1}(4)$.

(a) Let $y = \frac{x - 2}{6}$, then rearrange to make x the subject

$x = 6y + 2$, so $f^{-1}(x) = 6x + 2$

(b) $f^{-1}(4) = 26$

Answer: (a) $f^{-1}(x) = 6x + 2$ (b) 26

2 Averages with algebra

Here are five numbers: $3x + 1$, x, $x + 6$, $3x - 3$, $2x + 1$.

The mean of the five numbers is 15.

(a) Work out the value of x.

(b) Work out the median of the five numbers.

(a) Total = $(3x + 1) + (x) + (x + 6) + (3x - 3) + (2x + 1) = 15 \times 5 = 75$

$10x + 5 = 75$, so $10x = 70$ and $x = 7$

The numbers are 22, 7, 13, 18, 15

(b) In order: 7, 13, 15, 18, 22, so the median is 15

Answer: (a) x = 7 (b) 15

3 Rate of change of area or volume

A cylinder has radius r cm and fixed height 6 cm. Its volume is $V = 6\pi r^2$ cm^3.

(a) Find $\frac{dV}{dr}$ when r = 6.

(b) Find the value of r when $\frac{dV}{dr} = 24\pi$.

$\frac{dV}{dr} = 12\pi r$

When r = 6: $\frac{dV}{dr} = 72\pi$

$12\pi r = 24\pi$, so $12r = 24$ and $r = 2$ (lengths are positive)

Answer: (a) 72π (b) r = 2

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 43°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle APB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 43 = 86°$

(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angle APB = $360 - 90 - 90 - 86 = 94°$

Answer: (a) 86° (b) 94°

5 Simple and compound interest

Uma invests £4,000 for 5 years at 4% per year simple interest.

Lena invests £4,000 for 5 years at 4% per year compound interest.

Work out how much more interest Lena earns than Uma.

Give your answer to the nearest penny. (Calculator)

Simple interest: £4,000 × 4% × 5 = £800

Compound: £4,000 × 1.04^5 = £4,866.61

Compound interest = £4,866.61 − £4,000 = £866.61

Difference = £866.61 − £800 = £66.61

Answer: £66.61

6 Find unknown coefficients from gradients

The curve $y = x^3 + ax^2 + bx$ has gradient 0 when $x = 3$ and gradient -4 when $x = 1$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$

$x = 3$: $6a + b = -27$

$x = 1$: $2a + b = -7$

Subtract and solve: $a = -5$

$b = 3$

Answer: $a = -5$, $b = 3$

7 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 66°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 66°

Angle COB is an exterior angle of triangle OAC: 66 + 66 = 132°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 132°

8 Normal in the form ax + by + c = 0

The point A lies on the curve $y = 2x^2 - 6x - 6$ where $x = 1$.
Find the equation of the normal at A. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

$y = -10$ at $x = 1$ and gradient of curve $= -2$

Gradient of normal $= \frac{1}{2}$

$y + 10 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x - \frac{21}{2}$

Multiply by 2: $x - 2y - 21 = 0$

Answer: $x - 2y - 21 = 0$

9 Equal roots and real roots

The equation $kx^2 + 2x + 4 = 0$ has equal roots.
Find the value of k.

Equal roots means $b^2 - 4ac = 0$

$(2)^2 - 4 \times k \times 4 = 0$, so $4 = 16k$

$k = \frac{1}{4}$

Answer: $k = \frac{1}{4}$

10 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 5 cm.
The vertex V is 10 cm vertically above the centre O of the base.
M is the midpoint of BC.
Work out the angle between the face VBC and the base. (Calculator)

OM is perpendicular to BC, with OM = ½ × 5 = 2.5 cm

VM is perpendicular to BC, so the angle between the face and the base is angle VMO

$\tan VMO = \frac{VO}{OM} = \frac{10}{2.5}$

Angle = 76.0°

Answer: 76.0°