Day 361 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3p - q^3$
when $p = 3$, $q = 1$ and $r = -1$.

Substitute the values: $3 \times 3 - 1^3$
$= 9 - 1$
$= 8$
Answer: 8

2 Mean of a changing set

The mean of 7 numbers is 95.
One number is removed and the mean of the remaining 6 numbers is 108.
Work out the number that was removed.

Total of the 7 numbers = $7 \times 95 = 665$
Total of the 6 remaining = $6 \times 108 = 648$
Number removed = $665 - 648 = 17$
Answer: 17

3 Solving trigonometric equations

Solve $7 \tan x + 8 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = -\frac{8}{7}$

Related acute angle: $\tan^{-1}(\frac{8}{7}) = 48.8°$

tangent is negative in the 2nd and 4th quadrants, so $x = 180° - 48.8°$ and $x = 360° - 48.8°$
$x = 131.2°$ and $x = 311.2°$
Answer: $x = 131.2°$ or $x = 311.2°$

4 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$3x + 6y \leq 36$ and $4x + 2y \leq 36$.
Find the greatest value of $P = 9x + 7y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 6), (8, 2), (9, 0)
Values of P: (0, 0): 0, (0, 6): 42, (8, 2): 86, (9, 0): 81
Greatest value of P is 86 at (8, 2)
Answer: $P = 86$ when $x = 8$ and $y = 2$

5 Fencing with a partition

A rectangular enclosure is x metres wide and y metres long. It is fenced all round and divided into two equal parts by a fence each of length x metres, parallel to the width.
The total length of fencing used is 24 m.

(a) Show that the area of the enclosure is $A = -\frac{3}{2}x^2 + 12x$.

(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $3x + 2y = 24$, so $y = \frac{24 - 3x}{2}$

$A = xy = \frac{x(24 - 3x)}{2} = -\frac{3}{2}x^2 + 12x$

$\frac{dA}{dx} = -3x + 12 = 0$ gives $x = 4$

$\frac{d^2A}{dx^2} = -3 < 0$, so a maximum

$A = 24$
Answer: 24 m²

6 Standard form: adding and subtracting

Work out $6.8 \times 10^{-3} - 7.5 \times 10^{-6}$.
Give your answer in standard form.

Write both numbers with the same power of 10, 10^{-6}:
$6800 \times 10^{-6} - 7.5 \times 10^{-6}$
$= (6800 - 7.5) \times 10^{-6} = 6792.5 \times 10^{-6}$
$= 6.7925 \times 10^{-3}$
Answer: 6.7925×10^{-3}

7 Gradient with fractional or negative powers

The equation of a curve is $y = -6x^{3/2} + 4x - 2\sqrt{x}$.
Work out the gradient of the curve at the point where $x = 9$.

$\frac{dy}{dx} = -9\sqrt{x} + 4 - x^{-1/2}$

When $x = 9$: gradient $= -\frac{70}{3}$

Answer: $-\frac{70}{3}$

8 Surds with unknowns

Given that $(a + \sqrt{5})(b + \sqrt{5}) = 47 + 13\sqrt{5}$,
where a and b are positive integers with $a < b$, find the values of a and b.

$(a + \sqrt{5})(b + \sqrt{5}) = ab + (a + b)\sqrt{5} + 5$
Compare the $\sqrt{5}$ terms: $a + b = 13$
Compare the integer terms: $ab + 5 = 47$, so $ab = 42$
Two numbers with sum 13 and product 42 are 6 and 7
Answer: a = 6, b = 7

9 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 98°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: $360 - 90 - 90 - 98 = 82°$
(b) The angle at the circumference is half the angle at the centre: $82 \div 2 = 41°$
Answer: (a) 82° (b) 41°

10 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx$ has stationary points at $x = 0$ and $x = 4$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = 0$: $0 + 0a + b = 0$. $x = 4$: $48 + 8a + b = 0$
Subtract the equations: $-48 + -8a = 0$, so $a = -6$
Then $b = 0$
Answer: $a = -6$, $b = 0$