Day 360 **IGCSE 10-a-day: Model Solutions**

1 Expand then differentiate

$y = x(x - 4)(2x - 1)$

Find $\frac{dy}{dx}$.

Expand: $y = 2x^3 - 9x^2 + 4x$

$2x^3 \to 6x^2$, $-9x^2 \to -18x$, $4x \to 4$

$\frac{dy}{dx} = 6x^2 - 18x + 4$

Answer: $\mathbf{\frac{dy}{dx} = 6x^2 - 18x + 4}$

2 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$.

Find $|\mathbf{v}|$. Give your answer as a surd in its simplest form.

$|\mathbf{v}| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4 + 25}$

$= \sqrt{29}$

Answer: $\mathbf{\sqrt{29}}$

3 Inverse proportion in context

9 machines can make a batch of bolts in 12 hours.
After 5 hours, 2 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 9 × 12 = 108 machine-hours
Work done = 9 × 5 = 45, so work left = 63
Workers left = 9 − 2 = 7
Time = 63 ÷ 7 = 9 hours
Answer: 9 hours

4 Forming and solving quadratic equations

A rectangle has length (x + 6) cm and width (x + 4) cm.
The area of the rectangle is 99 cm^2.
(a) Show that $x^2 + 10x - 75 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(x + 6)(x + 4) = 99$, so $x^2 + 10x + 24 = 99$
$x^2 + 10x - 75 = 0$
(b) $x^2 + 10x - 75 = 0$: $(x - 5)(x + 15) = 0$
$x = 5$ ($x = -15$ is not possible as lengths are positive)
Length = 11 cm and width = 9 cm, so perimeter = 2 × (11 + 9) = 40 cm
Answer: (b) 40 cm

5 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 10: circumference = 2 × π × 10 = 20π
The two straight sides: 2 × 60 = 120
Boundary = (120 + 20π) m
(b) The rectangle is 60 m by 2 × 10 = 20 m, so its area is 60 × 20 = 1200. Circle: π × 10^2 = 100π
Area = (1200 + 100π) m^2
Answer: (a) (120 + 20π) m (b) (1200 + 100π) m^2

6 Mixtures and percentage profit

A shop mixes two types of mixed nuts.
Type A costs £6.00 per kg and type B costs £4.00 per kg.
They are mixed in the ratio A : B = 1 : 4.
The mixture is sold for £5.72 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 1 × £6.00 + 4 × £4.00 = £22.00
Cost per kg = £22.00 ÷ 5 = £4.40
Profit per kg = £5.72 − £4.40 = £1.32
Percentage profit = 1.32 ÷ 4.40 × 100 = 30%
Answer: 30%

7 Intersection and gradient

The curve $y = -2x^2 - 4x + 6$ and the line $y = -2x + 2$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$-2x^2 - 4x + 6 = -2x + 2$ gives $-2x^2 - 2x + 4 = 0$, so $-2(x + 2)(x - 1) = 0$
$x = -2$ or $x = 1$, giving A $(-2, 6)$ and B $(1, 0)$
$\frac{dy}{dx} = -4x - 4$: at A it is 4, at B it is -8
Answer: (a) A $(-2, 6)$, B $(1, 0)$ (b) 4 at A, -8 at B

8 Reverse percentages

The number of members of a club is increased by 10% in one year.
The next year the number of members is decreased by 10%.
There are now 891 members.
How many members were there to begin with?

Multipliers: 1.1, 0.9
Overall multiplier = 1.1 × 0.9 = 0.99
Original = final ÷ multiplier = 891 ÷ 0.99
= 900
Answer: 900

9 Tree diagrams and reverse conditional probability

Machine A makes 70% of the items and machine B makes the rest.
7% of items from A are faulty and 1% of items from B are faulty.
An item is chosen at random.
(a) Work out the probability that the item is faulty.
(b) The item is faulty. Work out the probability that it came from machine A.

(a) 0.7 × 0.07 = 0.049 and 0.3 × 0.01 = 0.003
P(faulty) = 0.049 + 0.003 = 0.052
(b) $\frac{0.049}{0.052} = \frac{49}{52}$
Answer: (a) 0.052 (b) $\frac{49}{52}$

10 Rectangle under a parabola

The diagram shows the curve $y = 108 - x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).

(a) Show that the area of ABCD is $-2x^3 + 216x$.

(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $108 - x^2$, so area = $2x(108 - x^2) = -2x^3 + 216x$

$\frac{dA}{dx} = -6x^2 + 216 = 0$, so $x^2 = 36$ and $x = 6$ $(x > 0)$

$\frac{d^2A}{dx^2} = -12x < 0$ when $x = 6$, so a maximum

Area = $216 \times 6 - 2 \times 6^3 = 864$

Answer: 864