Day 36 **IGCSE 10-a-day: Model Solutions**

1 Tangents parallel to a line

A curve has equation $y = x^3 - 3x^2 - 3x - 1$.
Find the equations of the two tangents to the curve that are parallel to the line $y = 6x - 1$.

Parallel lines have equal gradients: gradient = 6
$\frac{dy}{dx} = 3x^2 - 6x - 3 = 6$, so $3x^2 - 6x - 9 = 0$
$3(x + 1)(x - 3) = 0$ gives $x = -1$ or $x = 3$
At $x = -1$: $y = -2$, tangent $y - (-2) = 6(x - (-1))$
At $x = 3$: $y = -10$, tangent $y - (-10) = 6(x - 3)$
Answer: $y = 6x + 4$ and $y = 6x - 28$

2 Combined and changing means

The mean mark of 12 students in class A is 73.
The mean mark of 20 students in class B is 40.
Work out the mean mark of all 32 students.

Total marks for class A = $12 \times 73 = 876$
Total marks for class B = $20 \times 40 = 800$
Mean = $(876 + 800) \div 32 = 52.375$
Answer: 52.375

3 Equation of a normal

A curve has equation $y = 3x^2 + 2x - 6$.
Find the equation of the normal to the curve at the point where $x = 0$.

When $x = 0$, $y = -6$ and $\frac{dy}{dx} = 2$

Gradient of the normal = $-1 \div 2 = -\frac{1}{2}$

$y + 6 = -\frac{1}{2}x$

$y = -\frac{1}{2}x - 6$

Answer: $y = -\frac{1}{2}x - 6$

4 Solving trigonometric equations

Solve $\sin x = -0.5$ for $0 \leq x \leq 360$.
Give your answers correct to 1 decimal place. (Calculator)

$\sin x = -0.5$
Principal value: $x = \sin^{-1}(-0.5) = -30.0°$
Second solution: $x = 180° -$ (principal value)
All solutions in the range: 210.0, 330.0
Answer: x = 210.0, 330.0

5 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(3x + 5)°$, angle BCD = $(5x - 17)°$ and angle ABC = $(3x - 15)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(3x + 5) + (5x - 17) = 180$
$8x - 12 = 180$, so $8x = 192$
$x = 24$
(b) Angle ABC = $3 \times 24 - 15 = 57°$
Angle ADC = $180 - 57 = 123°$
Answer: (a) x = 24 (b) 123°

6 Equations in disguise

Solve $x^4 - 34x^2 + 225 = 0$

Let $u = x^2$: $u^2 - 34u + 225 = 0$
Factorise: $(u - 9)(u - 25) = 0$, so $u = 9$ or $u = 25$
$x^2 = 9$ or $x^2 = 25$
$x = \pm 3$ or $x = \pm 5$
Answer: $x = -5, -3, 3, 5$

7 Fencing with a partition

A rectangular pen is x metres wide and y metres long. It is fenced all round and divided into two equal parts by a fence each of length x metres, parallel to the width.
The total length of fencing used is 60 m.

(a) Show that the area of the pen is $A = -\frac{3}{2}x^2 + 30x$.

(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $3x + 2y = 60$, so $y = \frac{60 - 3x}{2}$

$A = xy = \frac{x(60 - 3x)}{2} = -\frac{3}{2}x^2 + 30x$

$\frac{dA}{dx} = -3x + 30 = 0$ gives $x = 10$

$\frac{d^2A}{dx^2} = -3 < 0$, so a maximum
$A = 150$
Answer: 150 m²

8 Bounds of speed, density and pressure

A train travels a distance of 72 km, correct to 2 significant figures.
The journey takes 7.6 hours, correct to 2 significant figures.
Work out the upper bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

$71.5 \leq$ distance < 72.5; $7.55 \leq$ time < 7.65
Upper bound of speed = biggest distance ÷ smallest time = 72.5 ÷ 7.55
= 9.60265... = 9.60 (3 s.f.)
Answer: 9.60 km/h

9 Ratio and mixtures

Container A holds juice and water in the ratio 3 : 4.
Container B holds juice and water in the ratio 5 : 2.
14 litres from container A and 21 litres from container B are mixed.
Work out the ratio of juice to water in the mixture.

From A: juice = $14 \times \frac{3}{7}$ = 6 litres, water = 8 litres

From B: juice = $21 \times \frac{5}{7}$ = 15 litres, water = 6 litres

juice: 6 + 15 = 21 and water: 8 + 6 = 14
21 : 14 = 3 : 2
Answer: 3 : 2

10 Area and quadratic equations

The diagram shows a rectangle of length (x + 2) cm, width (2x + 3) cm and area 120 cm^2.

(a) Show that $2x^2 + 7x - 114 = 0$.

(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(x + 2)(2x + 3) = 120$

Expand: $2x^2 + 7x + 6 = 120$, so $2x^2 + 7x - 114 = 0$

(b) Factorise: $(x - 6)(2x + 19) = 0$

$x = 6$ ($x = -\frac{19}{2}$ is not possible as a length would be negative)

Length = 8 cm and width = 15 cm

Diagonal = $\sqrt{8^2 + 15^2} = \sqrt{289}$ = 17.0 cm (3 s.f.)

Answer: (b) 17.0 cm