Day 359 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

Differentiate $-5x^2 - 2x - \frac{1}{x}$ with respect to x.

Write negative powers as indices: $y = -5x^2 - 2x - x^{-1}$

$-5x^2 \to -10x$, $-2x \to -2$, $-x^{-1} \to x^{-2}$

$\frac{dy}{dx} = -10x - 2 + x^{-2}$

Answer: $\mathbf{\frac{dy}{dx} = -10x - 2 + \frac{1}{x^2}}$

2 Equations with fractions

Solve $\frac{x-5}{8} + \frac{x+4}{2} = 2$

Multiply every term by 8: $(x - 5) + 4(x + 4) = 16$

Expand: $5x + 11 = 16$

$5x = 5$

$x = 1$

Answer: x = 1

3 Venn diagram: reading set notation

The Venn diagram shows information about 24 families.

A: have a garden B: have a car

(a) Find n(A′).

(b) Find n(A ∩ B′).

(a) n(A′) = 13 + 3 = 16

(b) n(A ∩ B′) = 5

Answer: (a) 16 (b) 5

4 Coordinates and gradient at the y-intercept

$y = 3x^4 - 3x^3 + 2x^2 + 3$

The curve crosses the y-axis at the point A.

Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 3, so A is (0, 3)

$\frac{dy}{dx} = 12x^3 - 9x^2 + 4x$

When x = 0, $\frac{dy}{dx} = 0$

Answer: A = (0, 3), gradient 0

5 Area of a segment

The circle has centre O and radius 15 cm. A and B are points on the circle and angle AOB = 110°. (Calculator)

Work out the area of the shaded segment. Give your answer correct to 3 significant figures.

The diagram is not drawn accurately.

Area of sector OAB = $\frac{110}{360} \times \pi \times 15^2 = 215.98$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 15 \times 15 \times \sin 110° = 105.72$ cm^2

Segment = sector − triangle = 215.98 − 105.72 = 110.27

= 110 cm^2 (3 s.f.)

Answer: 110 cm^2

6 Successive percentage change

The value of a car is decreased by 15%.
The new amount is then decreased by 20%.
Work out the overall percentage change.

Multipliers: 0.85 and 0.8
0.85 × 0.8 = 0.68
0.68 is a decrease of 32%
Answer: 32% decrease

7 Product of brackets: tangent gradient

$y = x(x - 2)(x - 2)$
Work out the gradient of the curve at the point where $x = -1$.

Expand: $y = x^3 - 4x^2 + 4x$
$\frac{dy}{dx} = 3x^2 - 8x + 4$
$x = -1$: $\frac{dy}{dx} = 15$
Answer: 15

8 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 65 − 45 = 20
IQR of B = 80 − 50 = 30
(b) Class B has the higher median (60 compared with 55), so on average Class B scored higher
Class A has the smaller interquartile range (20 compared with 30), so its marks are more consistent
Answer: (a) A: 20, B: 30 (b) B has the higher median, A has the smaller IQR

9 Counting factors

$N = 2 \times 3^3$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply
(1 + 1) × (3 + 1) = 2 × 4 = 8
Answer: 8

10 Extreme value of an expression in a region

x and y satisfy these inequalities:
$x \geq 4$, $y \geq 3$, $3x + 4y \leq 36$
Find the greatest possible value of 4x + 4y.

The region is a triangle. The extreme value occurs at a vertex.
Vertices: (4, 3), (8, 3), (4, 6)
Values of 4x + 4y: 28, 44, 40
Greatest value = 44
Answer: 44