Day 358 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3p - q^3$
when $p = -3$, $q = 1$ and $r = -3$.

Substitute the values: $3 \times (-3) - 1^3$
$= -9 - 1$
$= -10$
Answer: -10

2 Fraction problems

A bag contains sweets.
$\frac{1}{8}$ of the sweets are orange and $\frac{5}{8}$ of the sweets are lemon.
The rest of the sweets, 30 of them, are strawberry.
How many sweets are in the bag?

Fraction that are strawberry $= 1 - \frac{1}{8} - \frac{5}{8} = \frac{1}{4}$

$\frac{1}{4}$ of the total is 30

Total $= 30 \div \frac{1}{4} = 120$

Answer: 120

3 Tangent where the curve crosses the y-axis

The curve $y = 2x^2 + x - 4$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = -4$
$\frac{dy}{dx} = 4x + 1$, so the gradient at A is 1
Tangent: $y = x - 4$
At B, $y = 0$: $x - 4 = 0$, so $x = 4$
Answer: (4, 0)

4 Density of an alloy

An alloy contains 356 g of nickel and 142 g of zinc.
The density of nickel is 8.9 g/cm^3. The density of zinc is 7.1 g/cm^3.
Work out the density of the alloy.

Volume of nickel $= 356 \div 8.9 = 40$ cm^3
Volume of zinc $= 142 \div 7.1 = 20$ cm^3
Total mass = 498 g, total volume = 60 cm^3
Density $= 498 \div 60 = 8.3$ g/cm^3
Answer: 8.3 g/cm^3

5 Transformations of graphs

The graph of $y = f(x)$ has a maximum point at $(4, -2)$.
Write down the coordinates of the turning point of the graph of
(a) $y = f(x) + 3$
(b) $y = f(x/2) - 3$

(a) Start with the turning point $(4, -2)$
Apply the transformation to the point: $(4, 1)$
(b) $f(x/2)$ changes the x-coordinate to 8, and $f(x) - 3$ changes the y-coordinate to -5
Turning point: $(8, -5)$
Answer: (a) (4, 1) (b) (8, −5)

6 Image of a point under a graph transformation

The point P(−3, 4) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x) + 2
(b) y = f(x + 2)

(a) y = f(x) + 2: y is increased by 2, so P maps to (−3, 6)
(b) y = f(x + 2): x is decreased by 2, so P maps to (−5, 4)
Answer: (a) (−3, 6) (b) (−5, 4)

7 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx - 10$.
The curve has a turning point at M(−3, 8).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when x = −3, so b = 6a
The point (−3, 8) is on the curve: 9a − 3b − 10 = 8
Substitute b = 6a: −9a = 18, so a = −2 and b = −12
Answer: a = −2, b = −12

8 Growth and decay

A culture has 1000 bacteria.
The number of bacteria increases by 4% each hour.
After how many whole hours will there first be more than 1500 bacteria? (Calculator)

Multiplier each hour = 1.04
After 10 hours: $1000 \times 1.04^{10} = 1480$
After 11 hours: $1000 \times 1.04^{11} = 1539$
So the first time it is more than 1500 is after 11 hours
Answer: 11 hours

9 Reading a trigonometric graph

The diagram shows the graph of $y = a \cos bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 0, least value = −6
c = ½(0 + (−6)) = −3 and a = ½(0 − (−6)) = 3
One full cycle takes 180° (2 cycles between 0 and 360), so b = 360 ÷ 180 = 2
Answer: a = 3, b = 2, c = −3

10 Increasing and decreasing with powers

$f(x) = 3x + \frac{12}{x}$ for x > 0

(a) Find the range of values of x for which f is a decreasing function.
(b) Write down the least value of f(x).

$f'(x) = 3 - 12x^{-2} = 3 - \frac{12}{x^2}$

f is decreasing when $f'(x) < 0$: $3 < \frac{12}{x^2}$, so $x^2 < 4$

With x > 0 this gives 0 < x < 2
f decreases then increases, so the least value is at x = 2: f(2) = 12
Answer: (a) 0 < x < 2 (b) 12