**Day 357** **IGCSE 10-a-day: Model Solutions**

## 1 Time zones

A flight leaves London at 10:05 (London time).
It takes 5 hours 35 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 10:05 + 5 h 35 min = 15:40
Tokyo is 8 hours ahead of London, so add 8 hours
23:40
**Answer: 23:40 the same day**

## 2 Adding and scaling matrices

$A = \begin{pmatrix} 0 & -5 \\ -6 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & -4 \\ -5 & -6 \end{pmatrix}$.
Find the matrix X such that A + X = B.

X = B − A

$$X = \begin{pmatrix} -5 & -4 \\ -5 & -6 \end{pmatrix} - \begin{pmatrix} 0 & -5 \\ -6 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 1 \\ 1 & -10 \end{pmatrix}$$

**Answer:** $\mathbf{X = \begin{pmatrix} -5 & 1 \\ 1 & -10 \end{pmatrix}}$

## 3 Equation of a tangent

A curve has equation $y = x^2 + x - 4$.
Find the equation of the tangent to the curve at the point where x = 2.

When x = 2, y = 2
$\frac{dy}{dx} = 2x + 1$, so the gradient at x = 2 is 5
$y - 2 = 5(x - 2)$
$y = 5x - 8$
**Answer: y = 5x − 8**

## 4 Equal roots and real roots

The equation $x^2 + kx + 9 = 0$ has exactly one solution.
Find the possible values of k.
Give any surds in their simplest form.

One solution means $b^2 - 4ac = 0$
$k^2 - 4 \times 1 \times 9 = 0$, so $k^2 = 36$
$k = \pm\sqrt{36} = \pm 6$
**Answer: k = ±6**

## 5 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$$\frac{x}{\sin 68°} = \frac{13}{\sin 80°}$$

$$x = \frac{13 \times \sin 68°}{\sin 80°} = 12.239$$

x = 12.2 cm (1 d.p.)
**Answer: AC = 12.2 cm**

## 6 Average speed

A bus travels the first half of a journey at 30 km/h.
It travels the second half of the same journey at 90 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 90 km (a common multiple of 30 and 90)
Time for first half = 90 ÷ 30 = 3 hours
Time for second half = 90 ÷ 90 = 1 hours
Average speed = total distance ÷ total time = 180 ÷ 4 = 45 km/h
**Answer: 45 km/h**

## 7 Open box from a square sheet

Squares of side x cm are cut from the corners of a 12 cm square card sheet, which is folded into an open box of volume V $cm^3$.
(a) Show that $V = 4x^3 - 48x^2 + 144x$.
(b) Find x for the maximum volume, and this volume.

Base is a square of side $12 - 2x$, so $V = x(12 - 2x)^2 = x(144 - 48x + 4x^2) = 4x^3 - 48x^2 + 144x$
$\frac{dV}{dx} = 12x^2 - 96x + 144 = 0$, so $x^2 - 8x + 12 = 0$
$12(x - 2)(x - 6) = 0$, so $x = 2$ or $x = 6$
$x = 6$ is rejected (6 is more than half of 12). $\frac{d^2V}{dx^2} = 24x - 96$, which is $-48$ when $x = 2$, so a maximum
$V = 128$
**Answer: x = 2, V = 128 $cm^3$**

## 8 Similar shapes

Two cones, P and Q, are mathematically similar.
The volume of P is 16 $cm^3$ and the volume of Q is 54 $cm^3$.
The curved surface area of P is 16 $cm^2$.
Work out the curved surface area of Q.

Ratio of volumes = $16 : 54 = 2^3 : 3^3$
So the ratio of lengths is 2 : 3
Ratio of areas = $2^2 : 3^2$, so scale factor = $\frac{9}{4}$
$16 \times \frac{9}{4} = 36$ $cm^2$
**Answer: 36 $cm^2$**

## 9 Compound interest: find the rate

£1,000 is invested for 4 years at r% per year compound interest.
At the end of 4 years the investment is worth £1,310.80.
Work out the value of r. (Calculator)

$1000 \times (1 + \frac{r}{100})^4 = £1{,}310.80$
$(1 + \frac{r}{100})^4 = 1310.8 \div 1000 = 1.3108$
$1 + \frac{r}{100} = 1.07$
$r = 7$
**Answer: r = 7**

## 10 Increasing or decreasing at a point

$y = x^3 - 6x^2 + 9x$

At the point where $x = -1$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 3x^2 - 12x + 9$

When $x = -1$: $\frac{dy}{dx} = 24$

24 is positive, so the curve is increasing

**Answer: increasing (gradient 24)**