Day 356 **IGCSE 10-a-day: Model Solutions**

1 Depreciation

A van is bought for £25,000.
Its value falls by 20% each year.
Work out its value after 2 years. (Calculator)

Multiplier = $1 - 20 \div 100 = 0.8$

Value = 25000×0.8^2

= £16,000

Answer: £16,000

2 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 37 is taken. Work out how many come from Year 7.
(b) A different stratified sample has 27 from Year 10. Work out the size of this sample.

Total = 80 + 240 + 240 + 180 = 740

(a) Year 7: $\frac{80}{740} \times 37 = 4$

(b) Fraction sampled = $27 \div 180 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 740 = 111$

Answer: (a) 4 (b) 111

3 Determinant and inverse

$M = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$.

(a) Work out the determinant of M.
(b) Find M^{-1}.

(a) $\det M = 2 \times 2 - (-3) \times (-1) = 1$
(b) Swap the leading diagonal, change the signs of the other two entries, divide by the determinant

$M^{-1} = \frac{1}{1}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$

Answer: (a) 1 (b) $\begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}$

4 Fractional power functions

$f(x) = -2x^{5/2} + x - 6x^{1/2}$
Find $f'(x)$ and work out the value of $f'(9)$.

$-2x^{5/2} \to -5x^{3/2}$, $x \to 1$, $-6x^{1/2} \to -3x^{-1/2}$

$f'(x) = -5x^{3/2} + 1 - 3x^{-1/2}$

$f'(9)$: $\sqrt{9} = 3$, so $f'(9) = -135$

Answer: $f'(x) = -5x^{3/2} + 1 - 3x^{-1/2}$, $f'(9) = -135$

5 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-7 - 0}{0 - 3} = \frac{7}{3}$

Perpendicular gradient = $-1 \div \frac{7}{3} = -\frac{3}{7}$

M passes through B(3, 0): $0 = (-\frac{3}{7}) \times 3 + c$, so $c = \frac{9}{7}$

$y = -\frac{3}{7}x + \frac{9}{7}$

Answer: $y = -\frac{3}{7}x + \frac{9}{7}$

6 Exact lengths and areas

A right-angled triangle has base $(4 + \sqrt{11})$ cm and height $(4 - \sqrt{11})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area $= \frac{1}{2} \times (4 + \sqrt{11})(4 - \sqrt{11})$

$(4 + \sqrt{11})(4 - \sqrt{11}) = 16 - 11 = 5$

Area $= \frac{1}{2} \times 5 = \frac{5}{2}$

Answer: $\frac{5}{2}$ cm^2

7 Tangent meets the axes

The diagram shows the curve $y = x^2 - 2x + 6$ and the tangent at P(2, 6), meeting the axes at A and B.
(a) Find an equation of the tangent at P.
(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = 2x - 2$, so at $x = 2$ the gradient is 2

$y - 6 = 2(x - 2)$, so $y = 2x + 2$

B: $x = 0$ gives $y = 2$. A: $y = 0$ gives $x = -1$

Area = ½ × 1 × 2 = 1

Answer: (a) $y = 2x + 2$ (b) 1

8 Bounds of area and perimeter

A rectangle has length 21 cm and width 13 cm.
Both measurements are correct to the nearest centimetre.
Tom says that the perimeter of the rectangle could be exactly 65 cm.
Is Tom correct? You must show your working.

$20.5 \leq$ length < 21.5 and $12.5 \leq$ width < 13.5

Lower bound of perimeter $= 2 \times 20.5 + 2 \times 12.5 = 66$

Upper bound of perimeter $= 2 \times 21.5 + 2 \times 13.5 = 70$

So $66 \leq$ perimeter < 70

65 is outside this interval, so Tom is not correct

Answer: No, 65 is outside the interval

9 Tangent through a given point: find the constant

A curve has equation $y = x^2 + 5x + k$, where k is a constant.
The tangent to the curve at the point where $x = 2$ passes through $(-2, -19)$.
Find the value of k.

$\frac{dy}{dx} = 2x + 5$, so at $x = 2$ the gradient is 9

The point of tangency is $(2, 14 + k)$

Gradient between this point and $(-2, -19)$: $\frac{-19 - (14 + k)}{-2 - 2} = 9$

$-19 - (14 + k) = -36$

$k = 3$

Answer: $k = 3$

10 Enlargement

Triangle P has vertices A(−4, −2), B(−10, −8) and C(−10, 2).
Triangle Q has vertices A'(2, 4), B'(5, 7) and C'(5, 2).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −6, x-step from A' to B' = 3

Scale factor = 3 ÷ −6 = $-\frac{1}{2}$

The centre C satisfies: C + $-\frac{1}{2}$(A − C) = A', so (1 − $-\frac{1}{2}$)C = A' − $-\frac{1}{2}$ A

x: $\frac{3}{2}$ × x = 2 − ($-\frac{1}{2}$) × (−4) = 0, so x = 0

y: $\frac{3}{2}$ × y = 4 − ($-\frac{1}{2}$) × (−2) = 3, so y = 2

Answer: Enlargement, scale factor $-\frac{1}{2}$, centre (0, 2)