Day 355 **IGCSE 10-a-day: Model Solutions**

1 Exchange rates

Nadia changes £1100 into dollars at £1 = $1.2.
Nadia spends $1030 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Nadia get back?

£1100 × 1.2 = $1320
1320 − 1030 = $290
290 ÷ 1.25 = £232
Answer: £232

2 Recurring decimals

Convert the recurring decimal $0.54\dot{8}\dot{1}$ to a fraction.
Give your fraction in its simplest form.

Let x = 0.5481...
100x = 54.8181...
10,000x = 5481.8181...
Subtract: 9900x = 5427
$x = \frac{5427}{9900} = \frac{603}{1100}$
Answer: $\mathbf{\frac{603}{1100}}$

3 Equation of a tangent

A curve has equation $y = 2x^3 + 3x^2 + 1$.
Find the equation of the tangent to the curve at the point where x = 2.

When x = 2, y = 29
$\frac{dy}{dx} = 6x^2 + 6x$, so the gradient at x = 2 is 36
y − 29 = 36(x − 2)
y = 36x − 43
Answer: y = 36x − 43

4 Composite with an inverse function

f(x) = 4x − 3 and $g(x) = x^2 - 3x + 6$.
(a) Find $f^{-1}g(x)$.
(b) Work out $f^{-1}g(-1)$.

(a) $f^{-1}(x) = \frac{x + 3}{4}$
$f^{-1}g(x) = \frac{x^2 - 3x + 6 + 3}{4} = \frac{x^2 - 3x + 9}{4}$
(b) g(−1) = 10, so $f^{-1}g(-1) = \frac{10 + 3}{4}$
Answer: (a) $\mathbf{\frac{x^2 - 3x + 9}{4}}$ **(b)** $\mathbf{\frac{13}{4}}$

5 LCM problems

A red light flashes every 6 minutes.
A blue light flashes every 15 minutes.
A green light flashes every 24 minutes.
They all flash together at 9:00 am.
(a) At what time do they next flash together?

Find the LCM of 6, 15 and 24
$6 = 2 \times 3$, $15 = 3 \times 5$, $24 = 2^3 \times 3$
$LCM = 2^3 \times 3 \times 5 = 120$
120 minutes after 9:00 am is 11:00 am
Answer: 11:00 am

6 Find unknown coefficients from gradients

The curve $y = ax^2 + bx + 4$ has gradient -8 when $x = 0$ and gradient -16 when $x = -1$.
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b$

$x = 0$: $b = -8$

$x = -1$: $-2a + b = -16$

Subtract and solve: $a = 4$

$b = -8$

Answer: a = 4, b = −8

7 Histogram with unequal class widths

The histogram shows the ages, in years, of some visitors.
The bar for 20 to 40 represents 48 visitors.
(a) Work out the frequency of the class 70 to 80.
(b) Work out the total frequency.

Frequency = k × height × width. For 20 to 40: $48 = k \times 6 \times 20$ so $k = \frac{2}{5}$

(a) 70 to 80: $\frac{2}{5} \times 3 \times 10 = 12$

(b) Frequencies: 48, 2, 24, 42, 12

Total = 128

Answer: (a) 12 (b) 128

8 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 2 cm and base radius 4 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+9} = \frac{2}{4}$

$4h = 2(h + 9)$, so $2h = 18$ and $h = 9$

Large cone: $\frac{1}{3} \times \pi \times 4^2 \times 18 = 96\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 9 = 12\pi$

Frustum = $96\pi - 12\pi = 84\pi$ cm^3

Answer: 84π cm^3

9 Wire cut in two

A piece of thin metal rod 34 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is twice its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{(34 - 4x)^2}{18}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $34 - 4x = 6w$, so $w = \frac{34 - 4x}{6}$ and its area is $2w^2 = \frac{(34 - 4x)^2}{18}$

$A = x^2 + \frac{(34 - 4x)^2}{18} = \frac{17}{9}x^2 - \frac{136}{9}x + \frac{578}{9}$

$\frac{dA}{dx} = \frac{34}{9}x - \frac{136}{9} = 0$, so $x = 4$. $\frac{d^2A}{dx^2} = \frac{34}{9} > 0$, so a minimum

Square: 16, rectangle: $6 \times 3 = 18$, so $A = 34$

Answer: x = 4, least area 34 cm²

10 Algebraic proof (harder)

Prove that the sum of the squares of any two odd numbers is 2 more than a multiple of 8.

Let the odd numbers be $2a + 1$ and $2b + 1$.

$(2a + 1)^2 + (2b + 1)^2 = 4a^2 + 4a + 4b^2 + 4b + 2 = 4[a(a + 1) + b(b + 1)] + 2$

$a(a + 1)$ and $b(b + 1)$ are each a product of consecutive integers, so each is even

So $a(a + 1) + b(b + 1)$ is even, and $4 \times$ an even number is a multiple of 8

Hence the sum is 2 more than a multiple of 8

Answer: Proved