Day 354 **IGCSE 10-a-day: Model Solutions**

1 Multiplying 2 by 2 matrices

$A = \begin{pmatrix} 2 & -4 \\ -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -4 & 0 \end{pmatrix}$.

(a) Work out AB.

(b) Work out BA.

(a) AB: row 1 is $2 \times 1 + (-4) \times (-4)$ and $2 \times (-2) + (-4) \times 0$; row 2 is $(-4) \times 1 + 1 \times (-4)$ and $(-4) \times (-2) + 1 \times 0$

$AB = \begin{pmatrix} 18 & -4 \\ -8 & 8 \end{pmatrix}$

(b) $BA = \begin{pmatrix} 10 & -6 \\ -8 & 16 \end{pmatrix}$

Answer: (a) AB = $\begin{pmatrix} \mathbf{18} & \mathbf{-4} \\ \mathbf{-8} & \mathbf{8} \end{pmatrix}$ (b) BA = $\begin{pmatrix} \mathbf{10} & \mathbf{-6} \\ \mathbf{-8} & \mathbf{16} \end{pmatrix}$

2 Time zones

A flight leaves London at 20:20 (London time).

It takes 8 hours 30 minutes.

When it is 12:00 in London it is 17:00 in Delhi.

At what time, in Delhi time, does the flight land? Say if it is on a different day.

Arrival in London time = 20:20 + 8 h 30 min = 04:50 the next day

Delhi is 5 hours ahead of London, so add 5 hours

09:50 the next day

Answer: 09:50 the next day

3 Average and instantaneous velocity

A particle moves in a straight line. Its displacement from O after t seconds is s metres, where

$s = -t^2 - 6t + 7$

(a) Work out the average velocity of the particle between t = 3 and t = 9.

(b) Find the time at which the velocity equals this average velocity.

s(3) = −20 and s(9) = −128

Average velocity = $\frac{-128 - (-20)}{9 - 3} = -18$

$v = \frac{ds}{dt} = -2t - 6$

$-2t - 6 = -18$ gives t = 6

Answer: (a) −18 m/s (b) t = 6 s

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 72°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angles in quadrilateral OAPB: 360 − 90 − 90 − 72 = 108°

(b) The angle at the circumference is half the angle at the centre: 108 ÷ 2 = 54°

Answer: (a) 108° (b) 54°

5 Quadratic inequalities

Solve $2x^2 + 14x > 0$

Divide by 2: $x^2 + 7x > 0$

Solve $x^2 + 7x = 0$: $(x + 7)(x) = 0$, so x = −7 or x = 0

The graph is a U shape crossing the x-axis at −7 and 0; it is above the axis outside the roots

x < −7 or x > 0

Answer: x < −7 or x > 0

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6 Minimum point of y = ax + b/x

The diagram shows a sketch of the curve $y = 3x + \frac{27}{x}$ for $x > 0$.

The curve has a minimum point at M.

Find the coordinates of M, justifying that it is a minimum.

$y = 3x + 27x^{-1}$, so $\frac{dy}{dx} = 3 - 27x^{-2}$

$\frac{dy}{dx} = 0$: $3 = \frac{27}{x^2}$, so $x^2 = 9$ and $x = 3$ $(x > 0)$

$\frac{d^2y}{dx^2} = 54x^{-3} = 2 > 0$, so a minimum

$y = 9 + \frac{27}{3} = 18$

Answer: M (3, 18)

7 Histogram with unequal class widths

The histogram shows the masses, in kg, of some parcels.

The bar for 45 to 55 represents 16 parcels.

(a) Work out the frequency of the class 30 to 40.

(b) Work out the total frequency.

Frequency = k × height × width. For 45 to 55: $16 = k \times 4 \times 10$ so $k = \frac{2}{5}$

(a) 30 to 40: $\frac{2}{5} \times 6 \times 10 = 24$

(b) Frequencies: 4, 24, 14, 16

Total = 58

Answer: (a) 24 (b) 58

8 Relative speed

Kira sets off at 9:00 and cycles at 12 km/h along a path.

Callum sets off from the same place at 9:15 and cycles at 16 km/h along the same path.

At what time does Callum catch up with Kira?

When Callum starts, Kira is $0.25 \times 12 = 3$ km ahead

Callum gains $16 - 12 = 4$ km every hour

Time to catch up = $3 \div 4 = 0.75$ hours = 45 minutes

9:15 + 45 minutes = 10:00

Answer: 10:00

9 Similar shapes

Two cones, P and Q, are mathematically similar.

The volume of P is 108 cm^3 and the volume of Q is 32 cm^3.

The curved surface area of P is 81 cm^2.

Work out the curved surface area of Q.

Ratio of volumes = $108 : 32 = 3^3 : 2^3$

So the ratio of lengths is 3 : 2

Ratio of areas = $3^2 : 2^2$, so scale factor = $\frac{4}{9}$

$81 \times \frac{4}{9} = 36$ cm^2

Answer: 36 cm^2

10 Minimum gradient of a curve

A curve has equation $y = 2x^3 + 12x^2 + 25x + 3$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 6x^2 + 24x + 25$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $12x + 24 = 0$, so $x = -2$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is $12 > 0$), so this is a minimum

$\frac{dy}{dx} = 1$ when $x = -2$

Answer: minimum gradient 1 when $x = -2$