Day 353 **IGCSE 10-a-day: Model Solutions**

1 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-3\mathbf{a} + 5\mathbf{b}$, OQ = $3\mathbf{a} + 7\mathbf{b}$ and OR = $6\mathbf{a} + 8\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $6\mathbf{a} + 2\mathbf{b}$
QR = OR − OQ = $3\mathbf{a} + \mathbf{b}$

QR = $\frac{1}{2}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 1
Answer: (b) 2 : 1

2 Moving averages

The table shows the daily visitors over six periods.
The 3-point moving average centred on Wed is 19.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Thu.

(a) (14 + x + 29) ÷ 3 = 19
14 + x + 29 = 57, so x = 14
(b) (14 + 29 + 23) ÷ 3 = 66 ÷ 3 = 22
Answer: (a) x = 14 (b) 22

3 Minimum gradient of a curve

A curve has equation $y = x^3 + 9x^2 + 27x + 4$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 3x^2 + 18x + 27$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $6x + 18 = 0$, so $x = -3$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is 6 > 0), so this is a minimum

$\frac{dy}{dx} = 0$ when $x = -3$

Answer: minimum gradient 0 when $x = -3$

4 Composite functions in x

$f(x) = 2x + 5$ and $g(x) = \frac{-4}{x - 2}$, $x \neq 2$.

(a) Write fg(x) as a single fraction.
(b) Write gf(x) in its simplest form.

(a) $fg(x) = 2 \times \frac{-4}{x - 2} + 5 = \frac{-8}{x - 2} + 5$

$= \frac{5x - 18}{x - 2}$

(b) $gf(x) = g(2x + 5) = \frac{-4}{2x + 3}$

Answer: (a) $\frac{5x - 18}{x - 2}$ (b) $\frac{-4}{2x + 3}$

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
2, −1, −2, −1, 2
Find an expression for the nth term.

First differences: −3, −1, 1, 3
Second differences: 2, 2, 2, so a = 2 ÷ 2 = 1
Subtract n^2 from each term: 1, −5, −11, −17, −23
The remaining linear sequence has nth term −6n + 7
nth term = $n^2 - 6n + 7$
Answer: $n^2 - 6n + 7$

6 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, −2) lies on the curve and the gradient of the curve at this point is 14.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$
Point on curve: a + b = −2. Gradient: 2a − b = 14
Solving simultaneously gives a = 4 and b = −6
Answer: a = 4, b = −6

7 Iteration: rearranging equations

(a) Show that the equation $x^2 - 8x - 31 = 0$ has a solution between x = 10 and x = 11.

(b) Show that the equation can be rearranged to give $x = 8 + \frac{31}{x}$

(a) Let $f(x) = x^2 - 8x - 31$. $f(10) = 10^2 - 8 \times 10 - 31 = -11$ and $f(11) = 11^2 - 8 \times 11 - 31 = 2$
f(10) is negative and f(11) is positive. There is a change of sign, so there is a solution between 10 and 11
(b) Rearrange $x^2 - 8x - 31 = 0$: $x^2 - 8x = 31$
Factorise: $x(x - 8) = 31$
Divide by x: $x - 8 = \frac{31}{x}$, so $x = 8 + \frac{31}{x}$
Answer: Shown

8 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -t^3 + 15t^2 + t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -3t^2 + 30t + 1$
$a = \frac{dv}{dt} = -6t + 30$
Maximum v when a = 0: −6t + 30 = 0, so t = 5
$\frac{d^2v}{dt^2} = -6 < 0$, so v is a maximum
v = 76
Answer: 76 m/s when t = 5 s

9 Bounds of roots

The volume of a cube is 708 cm^3, correct to the nearest cm^3.
Work out the lower bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$707.5 \leq$ volume < 708.5

Lower bound of edge = $\sqrt[3]{707.5}$
= 8.91064... = 8.91 (3 s.f.)
Answer: 8.91 cm

10 Compound interest: find the rate

£5,000 is invested for 2 years at r% per year compound interest.
At the end of 2 years the investment is worth £5,832.
Work out the value of r. (Calculator)

$5000 \times (1 + \frac{r}{100})^2 =$ £5,832

$(1 + \frac{r}{100})^2 = 5832 \div 5000 = 1.1664$

$1 + \frac{r}{100} = 1.08$

$r = 8$

Answer: r = 8