Day 352 **IGCSE 10-a-day: Model Solutions**

1 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 7 cm, BC = 11 cm and CG = 8 cm.
(a) Work out the length of AG.
(b) Work out the size of the angle between AG and the face ABFE.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AG^2 = 7^2 + 11^2 + 8^2 = 234$, so AG = 15.30 cm
(b) G projects onto F on face ABFE (FG is perpendicular to the face), so the angle is GAF
$AF^2 = 7^2 + 8^2 = 113$ and FG = 11
$\tan GAF = \frac{11}{\sqrt{113}}$, angle = 46.0°
Answer: (a) 15.3 cm (b) 46.0°

2 Rate of change of area or volume

A cylinder has radius r cm and fixed height 5 cm. Its volume is $V = 5\pi r^2$ cm^3.
(a) Find $\frac{dV}{dr}$ when r = 2.
(b) Find the value of r when $\frac{dV}{dr} = 20\pi$.

$\frac{dV}{dr} = 10\pi r$
When r = 2: $\frac{dV}{dr} = 20\pi$
$10\pi r = 20\pi$, so 10r = 20 and r = 2 (lengths are positive)
Answer: (a) 20π (b) r = 2

3 Simplifying surds

Write $\sqrt{252} - \sqrt{112} + \sqrt{7} + \sqrt{63}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{252} = 6\sqrt{7}$; $\sqrt{112} = 4\sqrt{7}$; $\sqrt{63} = 3\sqrt{7}$
$(6 - 4 + 1 + 3)\sqrt{7} = 6\sqrt{7}$
Answer: $6\sqrt{7}$

4 Compound interest

£3,000 is invested for 2 years at 2.5% per year.
Work out how much more interest is earned with compound interest than with simple interest. (Calculator)

Simple interest = 3000 × 2.5 ÷ 100 × 2 = £150
Compound: 3000×1.025^2 = £3,151.88, so interest = £151.88
Difference = £151.88 − £150 = £1.88
Answer: £1.88

5 Tangent where the curve crosses the y-axis

The curve $y = -2x^2 - 3x - 3$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, x = 0 and y = −3
$\frac{dy}{dx} = -4x - 3$, so the gradient at A is −3
Tangent: y = −3x − 3
At B, y = 0: −3x − 3 = 0, so x = −1
Answer: (−1, 0)

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6 Recurring decimals

Show that $0.13\dot{8} = \frac{5}{36}$

Let x = 0.1388...
100x = 13.8888...
1000x = 138.8888...
Subtract: 900x = 125
$x = \frac{125}{900} = \frac{5}{36}$

Answer: x = $\frac{5}{36}$

7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (2, 32) and (4, 128).
Find the value of k and the value of a.

Substitute the points: $k a^2 = 32$ and $k a^4 = 128$

Divide: $a^2 = \frac{128}{32} = 4$

a = 2 (positive, since a > 0)
From the first point: k × 4 = 32, so k = 8
Answer: k = 8, a = 2

8 Tangent meets the curve again

A curve has equation $y = x^3 - 3x^2 + 3x + 2$.
The tangent to the curve at the point A, where x = 2, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 6x + 3$, so the gradient at A is 3, and A = (2, 4)
Tangent: y = 3x − 2
Equate: $x^3 - 3x^2 + 4 = 0$, which has a repeated root x = 2
Sum of the roots = 3, so 2 + 2 + x = 3 and x = −1
y = −5
Answer: B (−1, −5)

9 Cone made from a sector

A sector of a circle has radius 6 cm and angle 240°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{240}{360} \times 2\pi \times 6 = 8\pi$ cm
This is the circumference of the base: $2\pi r = 8\pi$, so r = 4 cm
(b) Slant height = 6 cm, so $\text{height}^2 = 6^2 - 4^2 = 20$, height = $2\sqrt{5}$ cm
Volume = $\frac{1}{3} \times \pi \times 4^2 \times 2\sqrt{5} = \frac{32\sqrt{5}}{3}\pi$ cm^3

Answer: (a) 4 cm (b) $\frac{32\sqrt{5}}{3}\pi$ cm^3

10 Venn diagrams with algebra

The Venn diagram shows how many of 29 students study French (F) or Spanish (S).

(a) Work out the value of x.

(b) A student is chosen at random from those who study Spanish. Work out the probability that this student also studies French.

(a) Total: $(x + 4) + x + (2x - 1) + 2 = 29$

$4x + 5 = 29$, so $x = 6$

(b) Students in the second circle: $x + 2x - 1 = 6 + 11 = 17$, of whom 6 are in both

Probability = $\frac{6}{17}$

Answer: (a) x = 6 (b) $\frac{6}{17}$