Day 351 **IGCSE 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 10 numbers is 28.
One number is removed and the mean of the remaining 9 numbers is 30.
Work out the number that was removed.

Total of the 10 numbers = 10 × 28 = 280
Total of the 9 remaining = 9 × 30 = 270
Number removed = 280 − 270 = 10
Answer: 10

2 Same gradient on two curves

Two curves have equations $y = 2x^2 + 2x$ and $y = 3x^2 + 8x + 2$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 4x + 2$

Second curve: $\frac{dy}{dx} = 6x + 8$

Equal gradients: $4x + 2 = 6x + 8$
$-2x = 6$, so $x = -3$
Answer: $x = -3$

3 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 40)°$, $(2x + 15)°$, $(2x - 15)°$, $(x + 10)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$7x + 50 = 540$, so $7x = 490$
$x = 70$
(b) The angles are 70°, 110°, 155°, 125°, 80°
The largest is 155°
Answer: (a) x = 70 (b) 155°

4 Completing the square

(a) Write $x^2 + 12x + 39$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 12x + 39$

(a) Half of 12 is 6, so $x^2 + 12x = (x + 6)^2 - 36$
$x^2 + 12x + 39 = (x + 6)^2 - 36 + 39$
$= (x + 6)^2 + 3$
(b) The minimum is when $x + 6 = 0$, so $x = -6$ and $y = 3$
Answer: (a) $(x + 6)^2 + 3$ (b) $(-6, 3)$

5 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$.
Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix} -8 \\ -12 \end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} + \begin{pmatrix} -8 \\ -12 \end{pmatrix} - \begin{pmatrix} -6 \\ -3 \end{pmatrix} = \begin{pmatrix} -6 \\ -14 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-6} \\ \mathbf{-14} \end{pmatrix}$

6 Tangent to a curve with negative or fractional powers

A curve has equation $y = -x^2 + \frac{6}{x}$.

Find the equation of the tangent to the curve at the point where $x = -2$.

$\frac{dy}{dx} = -2x - 6x^{-2}$

At $x = -2$: $y = -7$ and gradient $= \frac{5}{2}$

$y + 7 = \frac{5}{2}(x + 2)$

$y = \frac{5}{2}x - 2$

Answer: $y = \frac{5}{2}x - 2$

7 Exchange rates

Hassan changes £1200 into euros at £1 = €1.3.
Hassan spends €1180 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Hassan get back?

£1200 × 1.3 = €1560
1560 − 1180 = €380
380 ÷ 1.6 = £237.50
Answer: £237.50

8 Growth and decay

A car is worth £2,000 when new.
Its value decreases by 10% each year.
After how many whole years will it first be worth less than £500? (Calculator)

Multiplier each year = 0.9
After 13 years: £2,000 × 0.9^{13} = £508.37
After 14 years: £2,000 × 0.9^{14} = £457.54
So the first time it is less than £500 is after 14 years
Answer: 14 years

9 Open box of fixed surface area

An open tray has a square base of side x cm and height h cm. The total outside surface area is 108 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(108x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 108$, so $h = \frac{108 - x^2}{4x}$

$V = x^2h = \frac{x(108 - x^2)}{4} = \frac{1}{4}(108x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 27 = 0$, so $x^2 = 36$ and $x = 6$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -9 < 0$, so a maximum

$V = 108$
Answer: 108 cm^3

10 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 14 cm, BC = 6 cm, CD = 14 cm,
angle ABC = 89° and angle ACD = 49°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 14^2 + 6^2 - 2 \times 14 \times 6 \times \cos 89°$ so AC = 15.135 cm
Area ABC = ½ × 14 × 6 × sin 89° = 41.99
Area ACD = ½ × 15.135 × 14 × sin 49° = 79.96
Total area = 41.99 + 79.96 = 122

Answer: 122 cm^2