Day 350 **IGCSE 10-a-day: Model Solutions**

1 Distance-time graphs

The graph shows Hassan's cycle ride to a café and back.
(a) Work out Hassan's speed on the way back.
(b) Work out Hassan's average speed for the whole ride, including the stop.

(a) Return: 8 km in 1 hours, so speed = 8 ÷ 1 = 8 km/h
(b) Total distance = 2 × 8 = 16 km, total time = 2 hours
Average speed = 16 ÷ 2 = 8 km/h
Answer: (a) 8 km/h (b) 8 km/h

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x)°
Angle ABC = (x + 15)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x) + 2(x + 15) = 180
6x + 30 = 180, so 6x = 150
x = 25
(b) Angle BAC = 4 × 25 + 0 = 100°
Answer: (a) x = 25 (b) 100°

3 Line of best fit

The scatter graph shows distance from town (x) and price (y).
The line of best fit passes through (2, 34) and (6, 26).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{26 - 34}{6 - 2} = \frac{-8}{4} = -2$
y = −2x + c: using (2, 34), 34 = −2 × 2 + c, so c = 38
y = −2x + 38
(b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 2
Answer: (a) y = −2x + 38 (b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 2

4 Equation of a tangent

A curve has equation $y = x^2 - 6x + 1$.
Find the equation of the tangent to the curve at the point where x = −3.

When x = −3, y = 28
$\frac{dy}{dx} = 2x - 6$, so the gradient at x = −3 is −12
y − 28 = −12(x + 3)
y = −12x − 8
Answer: y = −12x − 8

5 Quadratic formula and surds

Solve $5x^2 - 6x - 4 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 5, b = −6, c = −4

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$x = \frac{6 \pm \sqrt{116}}{10}$$

$\sqrt{116} = 2\sqrt{29}$, so $x = \frac{6 \pm 2\sqrt{29}}{10}$

Divide the top and bottom by 2: $x = \frac{3 \pm \sqrt{29}}{5}$

Answer: $\mathbf{x = \frac{3 \pm \sqrt{29}}{5}}$

6 Variation: missing values

T is directly proportional to the square of m.
When m = 3, T = 18.
When m = 4, T = a.
When m = b, T = 200.
Work out the values of a and b.

$T = km^2$, $k = 18 \div 9 = 2$
$a = 2 \times 16 = 32$
$200 = 2b^2$ so $b^2 = 100$, b = 10
Answer: a = 32, b = 10

7 Gradient from a tangent

The diagram shows part of the curve $y = -2x^2 + 2x - 2$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

$$\text{Gradient} = \frac{0 - (-12)}{0 - (-2)} = 6$$

$$\frac{dy}{dx} = -4x + 2$$

$−4x + 2 = 6$, so $x = -1$
Answer: (a) 6 (b) x = −1

8 Collinear points with vectors

$\mathbf{OA} = -\mathbf{b}$, $\mathbf{OB} = 4\mathbf{a} + 3\mathbf{b}$ and $\mathbf{OC} = -2\mathbf{a} - 3\mathbf{b}$.
Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = 4\mathbf{a} + 4\mathbf{b}$
$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -2\mathbf{a} - 2\mathbf{b}$

$$\mathbf{AC} = -\frac{1}{2}\mathbf{AB}$$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: $\mathbf{AC} = -\frac{1}{2}\mathbf{AB}$**, so A, B, C are collinear**

9 Curves that touch

The curves $y = 2x^2 - 5x + 4$ and $y = 4x^2 - 17x + 22$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $2x^2 - 12x + 18 = 0$

That is $2(x - 3)^2 = 0$ with a repeated root $x = 3$

$y = 7$, so the point is (3, 7)

$\frac{dy}{dx} = 4x - 5$ on the first curve: gradient 7

Tangent: $y = 7x - 14$

Answer: (3, 7), $y = 7x - 14$

10 Proof with indices

Show that $3^{n+3} - 3^n$ is a multiple of 26 for all integer values of n.

$3^{n+3} = 3^n \times 3^3$

$3^{n+3} - 3^n = 3^n(3^3 - 1)$

$3^3 - 1 = 26$

$= 26 \times 3^n$, which is a multiple of 26

Answer: 26×3^n