Day 35 **IGCSE 10-a-day: Model Solutions**

1 Tangent to a factorised cubic

A curve has equation $y = (x + 3)(x - 3)(x + 3)$.
Find the equation of the tangent to the curve at the point where $x = -2$.

Expand: $y = x^3 + 3x^2 - 9x - 27$
$\frac{dy}{dx} = 3x^2 + 6x - 9$
At $x = -2$: $y = -5$ and gradient $= -9$
$y + 5 = -9(x + 2)$
$y = -9x - 23$
Answer: $y = -9x - 23$

2 Sine rule: finding an angle

Angle B is acute.
Work out the size of angle ABC.
Give your answer correct to 1 decimal place. (Calculator)

$\frac{\sin B}{AC} = \frac{\sin A}{BC}$
$\sin B = \frac{11.2 \times \sin 65°}{13.9}$
$B = \sin^{-1}(0.7303) = 46.9°$
Answer: 46.9°

3 Gradient with fractional or negative powers

The equation of a curve is $y = x^2 - \frac{12}{x}$.
Work out the gradient of the curve at the point where $x = 4$.

$\frac{dy}{dx} = 2x + 12x^{-2}$
When $x = 4$: gradient $= \frac{35}{4}$
Answer: $\frac{35}{4}$

4 Equations with algebraic fractions

Solve $\frac{5}{x - 5} - \frac{6}{x + 4} = \frac{2}{3}$

Multiply every term by $(x - 5)(x + 4)$: $5(x + 4) - 6(x - 5) = \frac{2}{3} \times (x - 5)(x + 4)$
Multiply by 3 to clear the fraction: $15(x + 4) - 18(x - 5) = 2(x - 5)(x + 4)$
Expand and rearrange so that one side is 0: $2x^2 + x - 190 = 0$
Factorise: $(x + 10)(2x - 19) = 0$
$x = -10$ or $x = \frac{19}{2}$
Answer: $x = -10$ or $x = \frac{19}{2}$

5 Percentages

A is 10% less than D.
D is 5% more than S.
Work out A as a percentage of S.

$A = 0.9 \times D$ and $D = 1.05 \times S$
$A = 0.9 \times 1.05 \times S = 0.945 \times S$
$0.945 \times 100 = 94.5\%$
Answer: 94.5%

6 Area of a triangle: ½ab sin C

The area of triangle ABC is 56.7 cm^2.
Work out the length of AC.
Give your answer correct to 3 significant figures. (Calculator)

Area = ½ × BC × AC × sin C
56.7 = ½ × 12.2 × AC × sin 134°

$$AC = \frac{113.4}{12.2 \times \sin 134°} = 12.9$$

Answer: 12.9 cm

7 Ratio and expressions

x : y = 6 : 5
Work out the ratio (4x − 2y) : (6x + 6y)
Give your answer in its simplest form.

x : y = 6 : 5, so let x = 6t and y = 5t
4(6t) − 2(5t) = 14t
6(6t) + 6(5t) = 66t
14t : 66t = 7 : 33
Answer: 7 : 33

8 Equation of a circle

A circle has centre C(−5, −4) and passes through the point P(0, −1).
Find an equation of the circle.

$r^2 = CP^2 = (0 - (-5))^2 + (-1 - (-4))^2$
$= 25 + 9 = 34$
$(x + 5)^2 + (y + 4)^2 = 34$
Answer: $(x + 5)^2 + (y + 4)^2 = 34$

9 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 1) cm, (x) cm and (2x − 1) cm. The longest side is (2x − 1) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 1)^2 + x^2 = (2x - 1)^2$
Expand and collect terms: $2x^2 - 6x = 0$
Factorise: 2x(x − 3) = 0
x = 3 or x = 0, and x must be positive, so x = 3
(b) The two shorter sides are 4 cm and 3 cm

$$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

Answer: (a) x = 3 (b) 6 cm^2

10 Where two tangents meet

The curve C has equation $y = 2x^2 + 3x - 5$.
The points A and B on C have x coordinates 1 and 5.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 4x + 3$. At A: gradient 7, A = (1, 0)
Tangent at A: y = 7x − 7. At B: gradient 23, B = (5, 60)
Tangent at B: y = 23x − 55
Equate: 7x − 7 = 23x − 55, so x = 3 and y = 14
Answer: T (3, 14)