Day 349 **IGCSE 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 5 numbers is 10.4.
A new number is added and the mean of all 6 numbers is 12.5.
Work out the new number.

Total of the 5 numbers = 5 × 10.4 = 52
New total = 6 × 12.5 = 75
New number = 75 − 52 = 23
Answer: 23

2 Express a vector in terms of two vectors

$\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -16 \\ -6 \end{pmatrix}$.
$\mathbf{c} = m\mathbf{a} + n\mathbf{b}$.
Find the value of m and the value of n.

3m − 2n = −16 and −2m − 2n = −6
Multiply the first by (−2) and the second by (−2): −6m + 4n = 32 and 4m + 4n = 12
Subtract: −10m = 20, so m = −2
Substitute back: n = 5
Answer: m = −2, n = 5

3 Where a line meets the axes

The line L has equation 8x + 15y = 120.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, y = 0: 8x = 120, so x = 15. A is (15, 0)
At B, x = 0: 15y = 120, so y = 8. B is (0, 8)
(b) $AB^2 = 15^2 + 8^2 = 289$
$AB = \sqrt{289} = 17$
Answer: (a) A(15, 0), B(0, 8) (b) 17

4 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 10 cm, top radius 2 cm and base radius 6 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+10} = \frac{2}{6}$
6 h = 2(h + 10), so 4 h = 20 and h = 5
Large cone: $\frac{1}{3} \times \pi \times 6^2 \times 15 = 180\pi$
Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 5 = \frac{20}{3}\pi$
Frustum = $180\pi - \frac{20}{3}\pi = \frac{520}{3}\pi$ cm^3
Answer: $\frac{520}{3}\pi$ cm^3

5 Increasing or decreasing at a point

$y = -x^3 - 3x^2 + 9x + 1$
At the point where x = −1, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = -3x^2 - 6x + 9$
When x = −1: $\frac{dy}{dx} = 12$
12 is positive, so the curve is increasing
Answer: increasing (gradient 12)

6 Compound interest

Priya puts £3,000 in a savings account.
Compound interest is paid at 6% in year 1, 5% in year 2 and 6% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.06, 1.05, 1.06
£3,000 × 1.06 × 1.05 × 1.06
= £3,539.34
Answer: £3,539.34

7 Vertical distance between stationary points

The curve $y = 2x^3 - 3x^2 - 12x + 1$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 - 6x - 12 = 6(x + 1)(x - 2)$
$x = -1$ and $x = 2$
At $x = -1$, $y = 8$; at $x = 2$, $y = -19$
Difference $= 8 - (-19) = 27$
Answer: 27

8 Exponential graph from points

The diagram shows part of the graph of $y = ab^x$.
The curve passes through (0, 81) and (2, 36).
(a) Find the values of a and b. (b) Work out the value of y when x = 4.

(a) At $x = 0$: $y = a \times b^0 = a$, so $a = 81$

At $x = 2$: $81 \times b^2 = 36$ so $b^2 = \frac{4}{9}$

$b = \frac{2}{3}$

(b) $y = 81 \times \left(\frac{2}{3}\right)^4 = 16$

Answer: (a) a = 81, b = $\frac{2}{3}$ (b) 16

9 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.
Over the two years the population increases by 21%.
Work out the value of x.

Yearly multiplier $= 1 + \frac{x}{100}$

Two years: $\left(1 + \frac{x}{100}\right)^2 = 1.21$

Take the square root: $1 + \frac{x}{100} = 1.1$

$\frac{x}{100} = 0.1$, so $x = 10$
Answer: x = 10

10 Three stationary points of a quartic

$y = -2x^4 + 4x^2 + 2$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = -8x^3 + 8x = -8x(x - 1)(x + 1)$

$\frac{dy}{dx} = 0$ gives $x = -1$, $x = 0$ or $x = 1$

Points: $(-1, 4)$, $(0, 2)$, $(1, 4)$

$\frac{d^2y}{dx^2} = -24x^2 + 8$

$x = -1$: $\frac{d^2y}{dx^2} = -16$; $x = 0$: $\frac{d^2y}{dx^2} = 8$; $x = 1$: $\frac{d^2y}{dx^2} = -16$

So $(-1, 4)$ is a maximum, $(0, 2)$ is a minimum, $(1, 4)$ is a maximum

Answer: $(-1, 4)$ maximum; $(0, 2)$ minimum; $(1, 4)$ maximum