**Day 348** **IGCSE 10-a-day: Model Solutions**

## 1 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = -x^5 + 5x^3 - 6x - 4$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$-x^5 \to -5x^4$, $5x^3 \to 15x^2$, $-6x \to -6$

$\frac{dy}{dx} = -5x^4 + 15x^2 - 6$

**Answer:** $\mathbf{\frac{dy}{dx} = -5x^4 + 15x^2 - 6}$

## 2 Solving AX = B

$A = \begin{pmatrix} 2 & -1 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by $A^{-1}$)

$A^{-1} = \frac{1}{6}\begin{pmatrix} 4 & 1 \\ 2 & 2 \end{pmatrix}$

$X = \frac{1}{6}\begin{pmatrix} 4 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

**Answer:** $\mathbf{X = \begin{pmatrix} 3 \\ 1 \end{pmatrix}}$

## 3 Geometric sequences

The 3rd term of a geometric sequence is 256.

The 5th term of the sequence is 1024.

All the terms of the sequence are positive.

(a) Work out the common ratio.

(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 1024 \div 256 = 4$

$r = \sqrt{4} = 2$ (positive, since all terms are positive)

(b) The 3rd term is $a \times r^2$: $a \times 4 = 256$

$a = 256 \div 4 = 64$

**Answer: (a) r = 2 (b) 64**

## 4 Simplifying surds

Write $\sqrt{96} - \sqrt{216} + \sqrt{150}$ in the form $a\sqrt{6}$

where a is an integer.

$\sqrt{96} = 4\sqrt{6}$; $\sqrt{216} = 6\sqrt{6}$; $\sqrt{150} = 5\sqrt{6}$

$(4 - 6 + 5)\sqrt{6} = 3\sqrt{6}$

**Answer:** $\mathbf{3\sqrt{6}}$

## 5 Stationary point of a quartic

$y = -2x^4 + 64x$

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -8x^3 + 64$

$\frac{dy}{dx} = 0$: $-8x^3 + 64 = 0$, so $x^3 = 8$

$x = 2$ and $y = 96$

$\frac{d^2y}{dx^2} = -24x^2$

At $x = 2$: $\frac{d^2y}{dx^2} = -96$, so a maximum

**Answer: (2, 96), maximum**

## 6 Direct proportion formulae

y is directly proportional to the cube of x.

When $x = 2$, $y = 32$.

(a) Find a formula for y in terms of x.

(b) Work out the positive value of x when $y = 108$.

(a) $y = kx^3$

$32 = k \times 8$, so $k = 32 \div 8 = 4$

$y = 4x^3$

(b) $108 = 4x^3$, so $x^3 = 108 \div 4 = 27$, giving $x = 3$

**Answer: (a) $y = 4x^3$ (b) $x = 3$**

## 7 Area and quadratic equations

The diagram shows a rectangle of length $(3x + 5)$ cm, width $(x + 2)$ cm and area 70 $cm^2$.

(a) Show that $3x^2 + 11x - 60 = 0$.

(b) Work out the perimeter of the rectangle.

(a) $(3x + 5)(x + 2) = 70$

Expand: $3x^2 + 11x + 10 = 70$, so $3x^2 + 11x - 60 = 0$

(b) Factorise: $(x - 3)(3x + 20) = 0$

$x = 3$ ($x = -\frac{20}{3}$ is not possible as a length would be negative)

Length = 14 cm and width = 5 cm

Perimeter = $2 \times (14 + 5) = 38$ cm

**Answer: (b) 38 cm**

## 8 Three stationary points of a quartic

$y = 2x^4 - 36x^2 + 7$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 72x = 8x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives $x = -3$, $x = 0$ or $x = 3$

Points: $(-3, -155)$, $(0, 7)$, $(3, -155)$

$\frac{d^2y}{dx^2} = 24x^2 - 72$

$x = -3$: $\frac{d^2y}{dx^2} = 144$; $x = 0$: $\frac{d^2y}{dx^2} = -72$; $x = 3$: $\frac{d^2y}{dx^2} = 144$

So $(-3, -155)$ is a minimum, $(0, 7)$ is a maximum, $(3, -155)$ is a minimum

**Answer: $(-3, -155)$ minimum; $(0, 7)$ maximum; $(3, -155)$ minimum**

## 9 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 3 : 5.
Tin B holds red paint and white paint in the ratio 5 : 5.
48 litres from tin A is mixed with some of tin B.
The ratio of red paint to white paint in the mixture is 11 : 15.
Work out how many litres were taken from tin B.

Let y litres be taken from B

red paint: $18 + \frac{5y}{10}$

white paint: $30 + \frac{5y}{10}$

Ratio 11 : 15, so $15(18 + \frac{5y}{10}) = 11(30 + \frac{5y}{10})$

$(2)y = 60$, so $y = 30$

**Answer: 30 litres**

## 10 Intersecting lines in a triangle

$\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OP} = \frac{3}{4}\mathbf{a}$ and $\mathbf{OQ} = \frac{1}{2}\mathbf{b}$.

AQ and BP meet at X.
(a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find the ratio AX : XQ.

Let $\mathbf{AX} = s\mathbf{AQ}$. Then $\mathbf{OX} = \mathbf{a} + s(-\mathbf{a} + \frac{1}{2}\mathbf{b}) = (1 - s)\mathbf{a} + \frac{1}{2}s\mathbf{b}$

Let $\mathbf{BX} = t\mathbf{BP}$. Then $\mathbf{OX} = \mathbf{b} + t(\frac{3}{4}\mathbf{a} - \mathbf{b}) = \frac{3}{4}t\mathbf{a} + (1 - t)\mathbf{b}$

Compare coefficients: $1 - s = \frac{3}{4}t$ and $\frac{1}{2}s = 1 - t$

Solving gives $s = \frac{2}{5}$ and $t = \frac{4}{5}$

(a) $\mathbf{OX} = \frac{3}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$

(b) AX = $\frac{2}{5}$ × AQ, so AX : XQ = $\frac{2}{5} : \frac{3}{5} = 2 : 3$

**Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$ (b) 2 : 3**