Day 347 **IGCSE 10-a-day: Model Solutions**

1 Turning points of a cubic

A curve has equation $y = x^3 - 27x - 1$.
Find the coordinates of the turning points of the curve. Use the second derivative to determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 - 27 = 3(x + 3)(x - 3)$

$\frac{dy}{dx} = 0$ gives $x = -3$ or $x = 3$

$y = 53$ when $x = -3$ and $y = -55$ when $x = 3$

$\frac{d^2y}{dx^2} = 6x$: at $x = -3$ it is -18 (maximum), at $x = 3$ it is 18 (minimum)

Answer: $(-3, 53)$ maximum, $(3, -55)$ minimum

2 Solving quadratics by factorising

Solve $x^2 - 4 = 3x$

Rearrange so that one side is 0: $x^2 - 3x - 4 = 0$
Factorise: $(x + 1)(x - 4) = 0$
$x + 1 = 0$ or $x - 4 = 0$
$x = -1$ or $x = 4$
Answer: $x = -1$ or $x = 4$

3 Distance-time graphs

The graph shows Zane's cycle ride to a café and back.
(a) Work out Zane's speed on the way back.
(b) Work out Zane's average speed for the whole ride, including the stop.

(a) Return: 60 km in 7.5 hours, so speed = 60 ÷ 7.5 = 8 km/h
(b) Total distance = 2 × 60 = 120 km, total time = 10 hours
Average speed = 120 ÷ 10 = 12 km/h
Answer: (a) 8 km/h (b) 12 km/h

4 Tangent where the curve crosses the y-axis

The curve $y = x^3 + 3x^2 + 3x - 3$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = -3$
$\frac{dy}{dx} = 3x^2 + 6x + 3$, so the gradient at A is 3
Tangent: $y = 3x - 3$
At B, $y = 0$: $3x - 3 = 0$, so $x = 1$
Answer: (1, 0)

5 Counting subsets

A set P has 512 subsets.
How many elements does P have?

A set with n elements has 2^n subsets
$2^n = 512 = 2^9$, so $n = 9$
Answer: 9

6 Bounds in geometry

A right-angled triangle has hypotenuse 17.8 cm and one other side 3.1 cm, both correct to 1 decimal place. (Calculator)
Work out the upper bound of the length of the third side. Give your answer correct to 3 significant figures.

The third side is $\sqrt{\text{hypotenuse}^2 - \text{side}^2}$
For the upper bound, use the upper bound of the hypotenuse and the lower bound of the other side
$\sqrt{17.85^2 - 3.05^2} = 17.6$ cm (3 s.f.)
Answer: 17.6 cm

7 Laws of indices

Write $\frac{4^{n+2} \times 2^{n-2}}{2^n}$ in the form 2^{an+c}
where a and c are integers.

$4 = 2^2$, $2 = 2^1$, $2 = 2^1$
$4^{n+2} = 2^{2(n+2)}$, $2^{n-2} = 2^{n-2}$, $2^n = 2^n$
Exponent: $2(n + 2) + (n - 2) - n = 2n + 2$
Answer: a = 2, c = 2

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 85$
$x + y = 13$

From the second equation, $y = -x + 13$
Substitute into the first equation: $x^2 + (-x + 13)^2 = 85$
Expand: $2x^2 - 26x + 84 = 0$
Divide by 2: $x^2 - 13x + 42 = 0$
Factorise: $(x - 6)(x - 7) = 0$, so $x = 6$ or $x = 7$
$y = -x + 13$ gives $y = 7$ and $y = 6$
Answer: x = 6, y = 7 and x = 7, y = 6

9 Describe a transformation from a diagram

Triangle T is mapped onto triangle T′ by a single transformation.
(a) Describe fully this transformation.
(b) Write down the matrix that represents it.

(a) Compare corresponding vertices, e.g. (2, 2) goes to (2, −2)
The transformation is a reflection in the x-axis
(b) The columns show where (1, 0) and (0, 1) go: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Answer: (a) Reflection in the x-axis (b) $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$

10 Distance travelled

The diagram shows the graph of $s = 2t^3 - 12t^2 + 18t$, the displacement in metres of a particle P from O after t seconds.
P changes direction at A and B. Work out the total distance travelled in the first 5 seconds.

$v = \frac{ds}{dt} = 6t^2 - 24t + 18 = 0$ at $t = 1$ and $t = 3$ (A and B)
$s(0) = 0$, $s(1) = 8$, $s(3) = 0$, $s(5) = 40$
Distances: $|8 - 0| + |0 - 8| + |40 - 0|$
$= 8 + 8 + 40 = 56$
Answer: 56 m