Day 346 **IGCSE 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 69°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 69°
Angle COB is an exterior angle of triangle OAC: 69 + 69 = 138°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 138°

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 25 is taken. Work out how many come from Group B.
(b) A different stratified sample has 14 from Group D. Work out the size of this sample.

Total = 600 + 150 + 150 + 350 = 1250

(a) Group B: $\frac{150}{1250} \times 25 = 3$

(b) Fraction sampled = 14 ÷ 350 = $\frac{1}{25}$, so sample size = $\frac{1}{25} \times 1250 = 50$

Answer: (a) 3 (b) 50

3 Bounds of speed, density and pressure

A train travels a distance of 920 km, correct to the nearest 10.
The journey takes 59 hours, correct to the nearest whole number.
Work out the upper bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

915 ≤ distance < 925; 58.5 ≤ time < 59.5
Upper bound of speed = biggest distance ÷ smallest time = 925 ÷ 58.5
= 15.8120... = 15.8 (3 s.f.)
Answer: 15.8 km/h

4 Composite functions in x

f(x) = 4x − 6 and g(x) = 2x − 6.
Show that fg(x) − gf(x) = −12.

fg(x) = 4(2x − 6) − 6 = 8x − 30
gf(x) = 2(4x − 6) − 6 = 8x − 18
fg(x) − gf(x) = −12
Answer: fg(x) − gf(x) = −12

5 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by
$v = 24t - 2t^2$ for $0 \le t \le 12$
Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -4t + 24$
Greatest speed when a = 0: −4t + 24 = 0, so t = 6
$\frac{d^2v}{dt^2} = -4 < 0$, so this is a maximum
$v = 24 \times 6 - 2 \times 6^2 = 72$
Answer: t = 6 s, greatest speed 72 m/s

6 Percentages of percentages

In a group of members, 70% are juniors and the rest are seniors.
25% of the juniors and 35% of the seniors play chess.
What percentage of all the members play chess?

Take 100 members: 70 are juniors and 30 are seniors
25% of 70 = 17.5 and 35% of 30 = 10.5
17.5 + 10.5 = 28 out of 100
Answer: 28%

7 Points with a given gradient on y = ax + b/x

A curve has equation $y = -4x - \frac{6}{x}$.

Find the coordinates of the points on the curve where the gradient is 2.

$\frac{dy}{dx} = -4 + \frac{6}{x^2}$

Set $\frac{dy}{dx} = 2$: $-6 = \frac{-6}{x^2}$

$x^2 = 1$, so $x = 1$ or $x = -1$
$y = -10$ when $x = 1$ and $y = 10$ when $x = -1$
Answer: (1, −10) and (−1, 10)

8 Variation: joint and combined

V is directly proportional to the square of h and directly proportional to r.
When h = 2 and r = 6, V = 240.
Find V when h = 5 and r = 4.

$V = kh^2r$
Substitute: 240 = k × 24 so k = 10
V = 10 × 100 = 1000
Answer: V = 1000

9 Algebraic proof (harder)

Prove algebraically that the sum of the cubes of any three consecutive integers is a multiple of 9.

$(n-1)^3 + n^3 + (n+1)^3 = 3n^3 + 6n = 3n(n^2 + 2)$
If n is a multiple of 3, then $3n(n^2 + 2)$ is a multiple of 9
If $n = 3m + 1$ or $n = 3m - 1$, then $n^2 + 2 = 9m^2 \pm 6m + 3 = 3(3m^2 \pm 2m + 1)$, a multiple of 3
In every case $3n(n^2 + 2)$ contains two factors of 3, so it is a multiple of 9
Answer: Proved

10 Normal to y = kx^2: show that

P is the point $(t, \frac{7}{2}t^2)$ on the curve $y = \frac{7}{2}x^2$, where $t \neq 0$.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{7}{2}t^2 + \frac{1}{7}$.

$\frac{dy}{dx} = 7x$, so the gradient of the tangent at P is 7t

Normal gradient $= -\frac{1}{7t}$

Normal: $y - \frac{7}{2}t^2 = -\frac{1}{7t}(x - t)$

Put x = 0: $y = \frac{7}{2}t^2 + \frac{t}{7t} = \frac{7}{2}t^2 + \frac{1}{7}$

Answer: shown: $y = \frac{7}{2}t^2 + \frac{1}{7}$