Day 345 **IGCSE 10-a-day: Model Solutions**

1 Solving quadratics by factorising

Solve $8x^2 + 5 = -22x$

Rearrange so that one side is 0: $8x^2 + 22x + 5 = 0$
Factorise: $(4x + 1)(2x + 5) = 0$
$4x + 1 = 0$ or $2x + 5 = 0$
$x = -\frac{1}{4}$ or $x = -\frac{5}{2}$

Answer: $x = -\frac{5}{2}$ or $x = -\frac{1}{4}$

2 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the masses, in kg, of 80 parcels.
(a) Find the median. (b) Find the interquartile range.
(c) How many parcels have a mass greater than 100?

(a) Median at cumulative frequency 40: reading across gives 100
(b) Lower quartile at 20 gives 80, upper quartile at 60 gives 140
IQR = 140 − 80 = 60
(c) At 100 the cumulative frequency is 40, so 80 − 40 = 40
Answer: (a) 100 (b) 60 (c) 40

3 Area of a triangle using sine

Triangle ABC has AB = 17 cm and BC = 4 cm.
The area of triangle ABC is 17 cm^2.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area $= \frac{1}{2} \times AB \times BC \times \sin B$

$17 = \frac{1}{2} \times 17 \times 4 \times \sin B$

$\sin B = 17 \div 34 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so $B = 180° - 30° = 150°$

Answer: 150°

4 Second derivative and nature at a given point

$y = x^3 + 6x^2 + 9x - 6$
Use the second derivative to show that the curve has a minimum at $x = -1$.

$\frac{dy}{dx} = 3x^2 + 12x + 9$

When $x = -1$: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = 6x + 12$

When $x = -1$: $\frac{d^2y}{dx^2} = 6$, which is positive

So there is a minimum at $x = -1$
Answer: minimum at $x = -1$

5 Sine rule: finding a side

Work out the length of AC.
Give your answer correct to 3 significant figures. (Calculator)

Angle B = 180° − 76° − 48° = 56°

$$\frac{AC}{\sin 56°} = \frac{17.4}{\sin 76°}$$

$$AC = \frac{17.4 \times \sin 56°}{\sin 76°} = 14.9$$

Answer: 14.9 cm

6 Greatest and least values on an interval

$f(x) = -x^3 + 3x^2 + 6$ for $-2 \leq x \leq 4$.
Find the greatest value and the least value of f(x) on this interval.

$f'(x) = -3x^2 + 6x = 0$ gives x = 0 or x = 2 (both in the interval)
f(−2) = 26, f(0) = 6, f(2) = 10, f(4) = −10
Compare the turning points and both end values
Answer: Greatest 26, least −10

7 Ratio and algebra

In a club, the ratio of the number of boys to the number of girls is 7 : 2.
Then 8 more boys are added and 16 more girls are added.
The ratio is now 2 : 1.
How many boys were there at the start?

Let the numbers at the start be 7k and 2k
(7k + 8) : (2k + 16) = 2 : 1, so 1(7k + 8) = 2(2k + 16)
7k + 8 = 4k + 32
3k = 24, so k = 8
Number of boys at the start: 7 × 8 = 56
Answer: 56

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 45$
x + y = 9

From the second equation, y = −x + 9
Substitute into the first equation: $x^2 + (-x + 9)^2 = 45$
Expand: $2x^2 - 18x + 36 = 0$
Divide by 2: $x^2 - 9x + 18 = 0$
Factorise: (x − 3)(x − 6) = 0, so x = 3 or x = 6
y = −x + 9 gives y = 6 and y = 3
Answer: x = 3, y = 6 and x = 6, y = 3

9 No stationary points

A curve has equation $y = x^3 + 3x^2 + 10x - 5$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$$\frac{dy}{dx} = 3x^2 + 6x + 10 = 3(x + 1)^2 + 7$$

A squared term is never negative, so $\frac{dy}{dx} \geq 7 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 7 (when x = −1)
Answer: (b) 7

10 Compound interest

Chen puts £6,000 in a savings account.
Compound interest is paid at 1% in year 1, 2% in year 2 and 5% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.01, 1.02, 1.05
£6,000 × 1.01 × 1.02 × 1.05
= £6,490.26
Answer: £6,490.26