**Day 344** **IGCSE 10-a-day: Model Solutions**

## 1 Time zones

A flight leaves London at 16:30 (London time).
It takes 8 hours 45 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 16:30 + 8 h 45 min = 01:15 the next day
New York is 5 hours behind London, so subtract 5 hours
20:15
**Answer: 20:15 the same day**

## 2 Exchange rates

Carlos changes £400 into euros at £1 = €1.35.
Carlos spends €120 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Carlos get back?

£400 × 1.35 = €540
540 − 120 = €420
420 ÷ 1.6 = £262.50
**Answer: £262.50**

## 3 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the opposite side BC, so use tan 45°
tan 45° = 1
x = 9 ÷ tan 45° = 9 ÷ 1
x = 9
**Answer: x = 9**

## 4 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.
$u_1 = 5$ and $u_2 = x$.
(a) Show that $u_5 = 3x + 10$.
(b) Given that $u_5 = 28$, find x and hence work out $u_8$.

(a) $u_3 = 5 + x$, $u_4 = u_3 + u_2 = 2x + 5$, $u_5 = u_4 + u_3 = 3x + 10$
(b) $3x + 10 = 28$ so $3x = 18$ and $x = 6$
Terms: 5, 6, 11, 17, 28, 45, 73, 118
$u_8 = 118$
**Answer: x = 6, $u_8$ = 118**

## 5 Tangents parallel to a line

A curve has equation $y = x^3 - 9x^2 + 20x - 3$.
Find the equations of the two tangents to the curve that are parallel to the line $y = 5x + 1$.

Parallel lines have equal gradients: gradient = 5
$\frac{dy}{dx} = 3x^2 - 18x + 20 = 5$, so $3x^2 - 18x + 15 = 0$
$3(x - 1)(x - 5) = 0$ gives $x = 1$ or $x = 5$
At x = 1: y = 9, tangent $y - 9 = 5(x - 1)$
At x = 5: y = −3, tangent $y - (-3) = 5(x - 5)$
**Answer: y = 5x + 4 and y = 5x − 28**

## 6 Algebraic proof

Prove algebraically that the sum of the squares of any five consecutive integers is always a multiple of 5.

Let the integers be $n - 2$, $n - 1$, $n$, $n + 1$ and $n + 2$.

$(n - 2)^2 + (n - 1)^2 + n^2 + (n + 1)^2 + (n + 2)^2 = 5n^2 + 2 \times 1 + 2 \times 4$

$= 5n^2 + 10 = 5(n^2 + 2)$

This is 5 × an integer, so it is a multiple of 5

**Answer: Proved**

## 7 Tangent through a given point: find the constant

A curve has equation $y = -x^2 - 4x + k$, where k is a constant.
The tangent to the curve at the point where $x = -3$ passes through (4, 16).
Find the value of k.

$\frac{dy}{dx} = -2x - 4$, so at $x = -3$ the gradient is 2

The point of tangency is $(-3, 3 + k)$

Gradient between this point and (4, 16): $\frac{16 - (3 + k)}{4 - (-3)} = 2$

$16 - (3 + k) = 14$

$k = -1$

**Answer: $k = -1$**

## 8 Angles of elevation from two points

Points A and B are on level ground, 118 m apart, with B between A and the foot of a flagpole.
The angle of elevation of the top of the flagpole is 42° from A and 71° from B.
Work out the height of the flagpole. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 71° − 42° = 29° (exterior angle of the triangle)

Sine rule in triangle ABT: $\frac{AT}{\sin 109°} = \frac{118}{\sin 29°}$

$AT = \frac{118 \times \sin 71°}{\sin 29°} = 230.13$

Height = AT × sin 42° = 154 m

**Answer: 154 m**

## 9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $\mathbf{a} + 5\mathbf{b}$, OQ = $3\mathbf{a} + 11\mathbf{b}$ and OR = $7\mathbf{a} + 23\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $2\mathbf{a} + 6\mathbf{b}$

QR = OR − OQ = $4\mathbf{a} + 12\mathbf{b}$

QR = 2 PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 2 : 4 = 1 : 2

**Answer: (b) 1 : 2**

## 10 Stationary points of y = ax + b/x

$y = -2x - \frac{32}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = -2 + \frac{32}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{32}{x^2} = 2$, so $x^2 = 16$ and $x = 4$ or $x = -4$

$y = -16$ when $x = 4$ and $y = 16$ when $x = -4$

$\frac{d^2y}{dx^2} = -\frac{64}{x^3}$

$x = 4$: $\frac{d^2y}{dx^2} = -1$ so maximum; $x = -4$: $\frac{d^2y}{dx^2} = 1$ so minimum

**Answer: (4, −16) maximum; (−4, 16) minimum**