Day 343 **IGCSE 10-a-day: Model Solutions**

1 Stationary point of a quartic

$y = 3x^4 - 12x$

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 12x^3 - 12$

$\frac{dy}{dx} = 0$: $12x^3 - 12 = 0$, so $x^3 = 1$

$x = 1$ and $y = -9$

$\frac{d^2y}{dx^2} = 36x^2$

At $x = 1$: $\frac{d^2y}{dx^2} = 36$, so a minimum

Answer: (1, −9), minimum

2 Enlargement

Triangle P has vertices A(2, 0), B(5, 3) and C(3, 2).
Triangle Q has vertices A'(1, −1), B'(7, 5) and C'(3, 3).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 3, x-step from A' to B' = 6
Scale factor = 6 ÷ 3 = 2
The centre C satisfies: C + 2(A − C) = A', so (1 − 2)C = A' − 2 A
x: (−1) × x = 1 − 2 × 2 = −3, so x = 3
y: (−1) × y = (−1) − 2 × 0 = −1, so y = 1
Answer: Enlargement, scale factor 2, centre (3, 1)

3 Inverse of a linear function

$f(x) = \frac{3x - 8}{3}$.

(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(-3)$.

(a) Let $y = \frac{3x - 8}{3}$, then rearrange to make x the subject

$x = \frac{3y + 8}{3}$, so $f^{-1}(x) = \frac{3x + 8}{3}$

(b) $f^{-1}(-3) = -\frac{1}{3}$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{3x + 8}{3}}$ (b) $\mathbf{-\frac{1}{3}}$

4 Reverse percentage growth and decay

A car loses 4% of its value each year.
After 3 years it is worth £24,330.24.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.96
Original × 0.96^3 = £24,330.24
Original = £24,330.24 ÷ 0.96^3 = £27,500
Answer: £27,500

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5 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.
The point P(3, 4) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{4-0}{3-0} = \frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{3}{4}$. Using P: $y - 4 = -\frac{3}{4}(x - 3)$

Multiply by 4: $4(y - 4) = -3(x - 3)$
Expand and rearrange: $3x + 4y = 25$
Answer: $3x + 4y = 25$

6 Solving trigonometric equations

Solve $2 \sin x - 2 = 0$ for $0 \leq x \leq 360$.

$\sin x = 1$
Principal value: $x = \sin^{-1}(1) = 90°$
Other solutions in the range: use symmetry of the sin graph
$x = 90$
Answer: $x = 90$

7 Open box of fixed surface area

An open tray has a square base of side x cm and height h cm. The total outside surface area is 12 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(12x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 12$, so $h = \frac{12 - x^2}{4x}$

$V = x^2h = \frac{x(12 - x^2)}{4} = \frac{1}{4}(12x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 3 = 0$, so $x^2 = 4$ and $x = 2$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -3 < 0$, so a maximum

$V = 4$
Answer: 4 cm^3

8 Error intervals

The height of a tower is 0.61 m, truncated to 2 decimal places.
Write down the error interval for the height of a tower, using h.

Truncating cuts off the later digits without rounding.
The smallest possible value is 0.61 and the value must be less than 0.62
$0.61 \leq h < 0.62$
Answer: $0.61 \leq h < 0.62$

9 Distance-time graphs

The graph shows Rhys's cycle ride to a café and back.
(a) Work out Rhys's speed on the way back.
(b) Work out Rhys's average speed for the whole ride, including the stop.

(a) Return: 8 km in 1 hours, so speed = 8 ÷ 1 = 8 km/h
(b) Total distance = 2 × 8 = 16 km, total time = 2.5 hours
Average speed = 16 ÷ 2.5 = 6.4 km/h
Answer: (a) 8 km/h (b) 6.4 km/h

10 Normal meets the x-axis

The diagram shows the curve $y = 2x^2 - 5x - 2$ and the normal at P(3, 1), meeting the x-axis at Q. R is (3, 0).
(a) Find the normal in the form $x + ky = c$.
(b) Work out the area of triangle PQR.

$\frac{dy}{dx} = 4x - 5$, so the gradient of the tangent at P is 7

Normal gradient = $-\frac{1}{7}$

$y - 1 = -\frac{1}{7}(x - 3)$. Multiply by 7: $x + 7y = 10$

Q: $y = 0$ gives $x = 10$, so Q = (10, 0) and QR = 7

Area = ½ × 7 × 1 = $\frac{7}{2}$

Answer: (a) $x + 7y = 10$ (b) $\frac{7}{2}$