Day 342 **IGCSE 10-a-day: Model Solutions**

1 Equation of a circle

A and B are the points (6, −7) and (−6, −5).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{6+(-6)}{2}, \frac{-7+(-5)}{2}) = (0, -6)$

$r^2 = (6-0)^2 + (-7-(-6))^2 = 36 + 1 = 37$

$x^2 + (y+6)^2 = 37$

Answer: $x^2 + (y+6)^2 = 37$

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x + 15)°
Angle PQR = (x + 25)°
Angle PRS = (4x + 49)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$4x + 49 = 5x + 40$
$x = 9$
(b) Angle PRS = 85° so angle PRQ = 180° − 85° = 95°
Answer: (a) x = 9 (b) 95°

3 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}$.
(a) Find M^{-1}.
(b) Use your answer to (a) to solve
3x + 3y = −3
3x + 4y = −7

(a) det M = 3 so $M^{-1} = \frac{1}{3}\begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} -3 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

Answer: (a) $\frac{1}{3}\begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix}$ (b) x = 3, y = −4

4 Local maximum and minimum values

$f(x) = 2x^3 - 6x + 3$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 6x^2 - 6 = 6(x-1)(x+1)$
$f'(x) = 0$ gives $x = -1$ or $x = 1$
$f''(x) = 12x$: $f''(-1) = -12 < 0$ (maximum) and $f''(1) = 12 > 0$ (minimum)
$f(-1) = 7$ and $f(1) = -1$
Answer: Local maximum y = 7, local minimum y = −1

5 nth term of a cubic sequence

Here are the first five terms of a sequence.
$-7, -1, 29, 95, 209$
The nth term is of the form $an^3 + bn + c$.
Find the values of a, b and c.

1st differences: 6, 30, 66, 114. 2nd: 24, 36, 48. 3rd: 12, 12
Third differences = $6a = 12$ so $a = 2$
Subtract $2n^3$: $-9, -17, -25, -33, -41$
This is linear with common difference -8, so $b = -8$, and $c = -9 - (-8) = -1$
Answer: a = 2, b = −8, c = −1

6 Points with a given gradient (cubic)

The curve $y = 2x^3 - 3x^2 - 47x + 1$ has gradient -11 at two points.
Find the coordinates of these points.

$\frac{dy}{dx} = 6x^2 - 6x - 47$
$6x^2 - 6x - 47 = -11$, so $6x^2 - 6x - 36 = 0$
Divide by 6: $x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$, so $x = -2$ or $x = 3$
Substitute into y: $(-2, 67)$ and $(3, -113)$
Answer: (−2, 67) and (3, −113)

7 Transformations of graphs

The graph of $y = f(x)$ has a minimum point at $(5, -3)$.
Write down the coordinates of the turning point of the graph of
(a) $y = 3f(x)$
(b) $y = f(-x) + 1$

(a) Start with the turning point $(5, -3)$
Apply the transformation to the point: $(5, -9)$
(b) $f(-x)$ changes the x-coordinate to -5, and $f(x) + 1$ changes the y-coordinate to -2
Turning point: $(-5, -2)$
Answer: (a) (5, −9) (b) (−5, −2)

8 Exact lengths and areas

A rectangle has area $(19 + 13\sqrt{5})$ cm^2 and length $(4 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{19 + 13\sqrt{5}}{4 + \sqrt{5}}$
Multiply top and bottom by $(4 - \sqrt{5})$:
Numerator: $(19 + 13\sqrt{5})(4 - \sqrt{5}) = 11 + 33\sqrt{5}$
Denominator: $16 - 5 = 11$
Width = $1 + 3\sqrt{5}$
Answer: $1 + 3\sqrt{5}$ cm

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to t.
When v = 2 and t = 4, P = 8.
Work out the value of P when v = 5 and t = 5.

$P = \frac{kv^2}{t}$

$8 = \frac{k \times 4}{4}$, so $k = 8 \times 4 \div 4 = 8$

$P = \frac{8 \times 25}{5} = 40$

Answer: P = 40

10 Where two tangents meet

The curve C has equation $y = x^2 - 4x + 6$.
The points A and B on C have x coordinates -1 and 3.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 2x - 4$. At A: gradient -6, A = $(-1, 11)$
Tangent at A: $y = -6x + 5$. At B: gradient 2, B = (3, 3)
Tangent at B: $y = 2x - 3$
Equate: $-6x + 5 = 2x - 3$, so $x = 1$ and $y = -1$
Answer: T (1, −1)